

On the effect of resonances in composite Higgs phenomenology

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Abstract

We consider a generic composite Higgs model based on the coset $SO(5)/SO(4)$ and study its phenomenology beyond the leading low-energy effective lagrangian approximation. Our basic goal is to introduce in a controllable and simple way the lowest-lying, possibly narrow, resonances that may exist in such models. We do so by proposing a criterion that we call *partial UV completion*. We characterize the simplest cases, corresponding respectively to a scalar in either singlet or tensor representation of $SO(4)$ and to vectors in the adjoint of $SO(4)$. We study the impact of these resonances on the signals associated to high-energy vector boson scattering, pointing out for each resonance the characteristic patterns of depletion and enhancement with respect to the leading-order chiral lagrangian. En route we derive the $O(p^4)$ general chiral lagrangian and discuss its peculiar accidental and approximate symmetries.

1 Introduction

It is rather plausible that a new strong dynamics be hiding behind electroweak symmetry breaking (EWSB). Unfortunately the modeling of the generic collider signals of such scenario is limited by our scarce ability to control strongly coupled quantum field theory. In particular, while in the low-energy regime effective chiral lagrangians provide in principle a reliable and universal description of the dynamics, it is in the physics of the massive states that strong coupling and model dependence represent a real limitation. Moreover, the separation of energy scales is often not enough in practice, so that the corrections to the simple chiral lagrangian results can be important. The study of WW scattering with realistic cuts (to beat the Standard Model (SM) background) is an example of that. Modeling the effects of heavy resonances, or at least assess their qualitative effects is thus a very relevant phenomenological problem. Some progress has come in recent years from extra dimensional constructions. The resulting models can be viewed as *deformations* of strongly coupled field theories where a parameter, basically the inverse volume of compactification, controls a weak coupling expansion. The couplings of the resonances are thus controlled to some extent. However, the studies in the literature are often made complicated by the presence of more structure and more parameters than one would hope to need in order to parametrize the physics of the low-lying resonances. In order to best serve the needs of the experimental community it would be preferable to introduce as simple as possible a description without sacrificing too much theoretical consistency. That is the main goal of the present paper. More precisely, we will be focussing on the scenario where the Higgs boson is a pseudo Nambu-Goldstone (NG) boson resulting from the coset $SO(5)/SO(4)$ [1, 2], and study the impact of different resonances on the scattering of NG bosons (longitudinal vector bosons plus the Higgs boson). Our study is very much in line with the recent effort to construct “simplified models” to describe the broad LHC signatures of classes of models [3]. Among previous studies that are related to ours, we should mention Refs. [4, 5, 6, 7] and the more recent [8, 9, 10, 11], where the impact of resonances in WW scattering for the Higgsless $SO(4)/SO(3)$ case was studied. Closer perhaps to our approach is Ref. [12], where two- and three-site deconstructed models for $SO(5)/SO(4)$ were introduced. In fact, our approach is even more minimal, as we are not attempting to parametrize the physics that gives rise to the Higgs potential, but just focussing on the scattering of NG bosons. Moreover, as (preliminary) part to our study of resonances we will construct the subleading $O(p^4)$ effective chiral lagrangian for the $SO(5)/SO(4)$ coset. That will allow us to compare to the specific $O(p^4)$ effects mediated by the exchange of each specific resonance. We will also describe in details the peculiar pattern of accidental and approximate discrete symmetries that arise in general in the models based on $SO(5)/SO(4)$.

2 The $SO(5)/SO(4)$ chiral lagrangian at $O(p^4)$

A systematic method to construct the chiral lagrangian for a generic Lie group \mathcal{G} spontaneously broken to a subgroup \mathcal{H} has been introduced by Callan, Coleman, Wess and Zumino (CCWZ) [13].¹ Their main result can be summarized as follows: the most general la-

¹ The results of Ref. [13] apply to any compact, connected, semisimple Lie group \mathcal{G} .

grangian invariant under global \mathcal{G} transformations can be built following the rules for a *local* symmetry \mathcal{H} , where \mathcal{H} is the linearly-realized subgroup of \mathcal{G} , by means of suitably defined covariant variables. Here we apply the CCWZ formalism to construct the $SO(5)/SO(4)$ chiral lagrangian at order $O(p^4)$.

Let us start by considering a generic global symmetry $\mathcal{G} \rightarrow \mathcal{H}$. We denote the Nambu-Goldstone fields by $\Pi(x) = \Pi^{\hat{a}}(x)T^{\hat{a}}$ and define

$$U(\Pi) = e^{i\Pi(x)} \quad (1)$$

$$-iU^\dagger \partial_\mu U = d_\mu^{\hat{a}} T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu. \quad (2)$$

Here and in the following $T^{\hat{a}}$, T^a are respectively the broken and unbroken generators normalized as $\text{Tr}(T^A T^B) = \delta^{AB}$ (the explicit expressions for the $SO(5)$ generators can be found in Appendix A). The action of $g \in \mathcal{G}$ on the NG fields is given by

$$gU(\Pi) \equiv U(g(\Pi))h(\Pi, g), \quad (3)$$

where $h(\Pi(x), g)$ is an element of \mathcal{H} which depends on the space-time point through $\Pi(x)$, and $U(g(\Pi)) = \exp(iT^{\hat{a}}\xi^{\hat{a}}(\Pi, g))$. Hence, under global \mathcal{G} transformations

$$U(\Pi) \rightarrow gU(\Pi)h^\dagger(\Pi, g), \quad (4)$$

which implies

$$d_\mu(\Pi) \rightarrow h(\Pi, g)d_\mu(\Pi)h^\dagger(\Pi, g) \quad (5)$$

$$E_\mu(\Pi) \rightarrow h(\Pi, g)E_\mu(\Pi)h^\dagger(\Pi, g) - ih(\Pi, g)\partial_\mu h^\dagger(\Pi, g). \quad (6)$$

The above equations show that $d_\mu(\Pi)$, $E_\mu(\Pi)$ transform under a local symmetry \mathcal{H} , and that in particular $E_\mu(\Pi)$ transforms like a gauge field. It is thus possible to define a covariant derivative

$$\nabla_\mu \equiv \partial_\mu + iE_\mu \quad (7)$$

and a field strength

$$E_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu + i[E_\mu, E_\nu] \quad (8)$$

$$E_{\mu\nu}(\Pi) \rightarrow h(\Pi, g)E_{\mu\nu}(\Pi)h^\dagger(\Pi, g).$$

The covariant variables $d_\mu(\Pi)$, $E_{\mu\nu}(\Pi)$, and those formed by acting with the covariant derivative ∇_μ are the building blocks of the chiral lagrangian.

There are additional covariant structures, however, that can be constructed once the external gauging of a group $\mathcal{H}' \subseteq \mathcal{G}$ is turned on (eventually, we will be interested in gauging a subgroup $SU(2)_L \times U(1)_Y$ identified with the SM electroweak group). In this case all the above relations hold true provided one replaces $\partial_\mu \rightarrow D_\mu = \partial_\mu + iA_\mu$ in eq.(2), which thus becomes

$$-iU^\dagger D_\mu U = d_\mu^{\hat{a}} T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu. \quad (9)$$

The external gauge fields $A_\mu = A_\mu^{\hat{a}} T^{\hat{a}} + A_\mu^a T^a$ transform according to the usual rule under (local) \mathcal{H}' transformations:² $A_\mu \rightarrow gA_\mu g^\dagger - ig\partial_\mu g^\dagger$. Two new covariant structures (i.e.

²To assign quantum numbers we formally take A to gauge the whole G .

variables that transform as representations of the local symmetry \mathcal{H}) can thus be constructed from the field strength of the external gauge fields as follows:

$$f_{\mu\nu} = U^\dagger F_{\mu\nu} U = (f_{\mu\nu}^-)^{\hat{a}} T^{\hat{a}} + (f_{\mu\nu}^+)^a T^a \equiv f_{\mu\nu}^- + f_{\mu\nu}^+ \quad (10)$$

$$f_{\mu\nu}^\pm(\Pi) \rightarrow h(\Pi, g) f_{\mu\nu}^\pm(\Pi) h^\dagger(\Pi, g).$$

2.1 The $SO(5)/SO(4)$ chiral lagrangian at $O(p^2)$ and its accidental symmetries

The lagrangian of composite Higgs models based on the $SO(5)/SO(4)$ coset can be easily constructed by means of the CCWZ covariant variables defined above. In this case there are four NG bosons associated to the breaking $SO(5) \rightarrow SO(4)$, $\pi^{\hat{a}}$ with $\hat{a} = 1, 2, 3, 4$, which live on the four-sphere ($SO(5)/SO(4) = S^4$). They transform as a **4** of $SO(4)$, or equivalently as a **(2, 2)** of $SU(2) \times SU(2) \sim SO(4)$. The SM electroweak vector bosons gauge a subgroup $SU(2)_L \times U(1)_Y \subset SU(2)_L \times SU(2)_R \sim SO(4)'$ contained in $SO(5)$, such that $Y = T_{3R}$.³ It is possible to parametrize the orientation of the ‘gauged’ $SO(4)'$ (i.e. that which contains the SM group $SU(2)_L \times U(1)_Y$) with respect to the linearly-realized global $SO(4)$ by an angle θ . For example, by representing the vacuum as a 5-dimensional unit vector Φ_0 , and letting the gauged $SO(4)'$ act on the first four entries, one has $\Phi_0 = (0, 0, 0, \sin \theta, \cos \theta)$. The gauged $SO(4)'$ thus identifies a preferred direction inside $SO(5)$, and the angle θ precisely measures the misalignment of the vacuum with respect to it, see Fig. 1. The field

$$\Phi = U(x)\Phi_0 = e^{i\sqrt{2}T^{\hat{a}}(\theta)\pi^{\hat{a}}(x)/f}\Phi_0 = \begin{pmatrix} \hat{\pi}^1 \sin(\pi/f) \\ \hat{\pi}^2 \sin(\pi/f) \\ \hat{\pi}^3 \sin(\pi/f) \\ \hat{\pi}^4 \sin(\pi/f) \cos \theta + \cos(\pi/f) \sin \theta \\ -\hat{\pi}^4 \sin(\pi/f) \sin \theta + \cos(\pi/f) \cos \theta \end{pmatrix} \quad (11)$$

parametrizes the massless excitations around the vacuum, where we have defined $\pi = \sqrt{(\pi^{\hat{a}})^2}$ and $\hat{\pi}^{\hat{a}} = \pi^{\hat{a}}/\pi$.⁴ Here and in the following we denote the generators of $SO(5) \rightarrow SO(4)$ as $T^{a,\hat{a}} = T^{a,\hat{a}}(\theta)$, which are related to those of $SO(5) \rightarrow SO(4)'$, where $SO(4)'$ is the gauged subgroup, by a rotation of an angle θ , see Appendix A.

For $\theta = 0$ the SM electroweak group is unbroken, being contained in the preserved global $SO(4)$, and the four NG bosons form a complex doublet of $SU(2)_L$. For $\theta \neq 0$, on the other hand, the SM vector bosons gauge (a combination of) the $SO(5)/SO(4)$ broken generators, so that three NG bosons are eaten to give mass to the W and the Z , while a fourth one is identified with the Higgs boson. This can be easily seen as follows. Since the gauged $SO(4)'$ acts on the first four entries of the field Φ in eq.(11), these can be conveniently

³In realistic models there is a larger pattern of global symmetries, $SO(5) \times U(1)_X \rightarrow SO(4) \times U(1)_X$, and hypercharge is defined as $Y = T_{3R} + X$. A non-zero X charge is required for the SM fermions to correctly reproduce their hypercharge. Since the NG bosons are neutral under the additional $U(1)_X$, this latter plays no role in the following discussion and will be omitted for simplicity.

⁴The factor $\sqrt{2}$ in the exponent of eq.(11) has been introduced to match the standard normalization adopted in the literature. It can be absorbed by a redefinition of f .

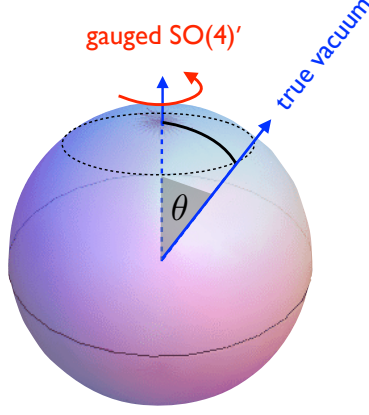


Figure 1: The NG bosons of $SO(5)/SO(4)$ live on the four-sphere S^4 . A generic vacuum points in a direction forming an angle θ with that fixed by the ‘gauged’ $SO(4)'$. The electroweak symmetry breaking can be seen as due to the misalignment θ . Even assuming no misalignment at the tree level, a non-vanishing $\theta = \langle \pi \rangle / f$ is generated at the loop level after the NG 4-vector acquires a vev $\langle \pi \rangle \neq 0$ (black curve).

rewritten as a modulus, ϕ_4 , times a unit 4-vector. The unit vector can in turn be expressed as a constant vector invariant under electromagnetic ($U(1)_{em}$) transformations times a phase $\exp(i\chi^i(x)A^i/v)$, where A^i are $SO(4)'/SO(3)$ generators. Considering that $\|\Phi\| = 1$ implies $\phi_4 \leq 1$, and that in the vacuum $\langle \phi_4 \rangle = \sin \theta$, it is convenient to define $\phi_4(x) \equiv \sin(\theta + h(x)/f)$. Hence,

$$\Phi = \begin{pmatrix} \sin(\theta + h(x)/f) e^{i\chi^i(x)A^i/v} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\ \cos(\theta + h(x)/f) \end{pmatrix}. \quad (12)$$

By construction, the three χ^i are the fields eaten after the $SU(2)_L \times U(1)_Y$ external gauging is turned on, while h , which parametrizes $SO(4)'$ -invariant fluctuations around the vacuum θ , remains in the spectrum as a pseudo-NG boson. It is thus identified with the Higgs boson. By equating (11) and (12) one obtains the (non-linear) field redefinition that relates the four NG bosons of $SO(5)/SO(4)$, $\pi^{\hat{a}}$, and the ‘physical’ degrees of freedom, χ^i , h :

$$\begin{aligned} \sin(\theta + h(x)/f) \hat{\chi}^i(x) \sin(\chi(x)/v) &= \hat{\pi}^i(x) \sin(\pi(x)/f), & i = 1, 2, 3 \\ \cos(\theta + h(x)/f) &= \cos(\pi(x)/f) \cos \theta - \hat{\pi}^4(x) \sin(\pi(x)/f) \sin \theta, \end{aligned} \quad (13)$$

where $\chi \equiv \sqrt{(\chi^i)^2}$, $\hat{\chi}^i \equiv \chi^i/\chi$.

In realistic models, the value of θ is dynamically determined, and the breaking of the electroweak symmetry can be seen as the result of a vacuum misalignment. Another point of view, however, is possible and sometimes useful. If all the explicit breaking of the global $SO(5)$ comes from the $SU(2)_L \times U(1)_Y$ external gauging and from the couplings of other elementary fields (in particular the SM fermions), then at tree level the orientation of the

vacuum is arbitrary and one can suitably set $\theta = 0$ (so that $SO(4)' = SO(4)$). With this choice, the four NG bosons of $SO(5)/SO(4)$ transform as a complex doublet of the gauged $SU(2)_L$, and none of them is eaten. Loop corrections will however generate a potential for the NG bosons and can lead to a non-vanishing vev for the modulus of the NG 4-vector: $\langle\pi\rangle \neq 0$ (see Fig. 1). As a result, $SO(4)$ is spontaneously broken to (a custodial) $SO(3)$, and three of the original NG bosons are eaten. The field Φ can be recast in the form of eq.(12) by identifying $\theta = \langle\pi\rangle/f$ and the field $h(x)$ as the fluctuation of the modulus of the NG 4-vector around its vev. One can thus think of the electroweak symmetry breaking as a two-step process: a first spontaneous breaking, $SO(5) \rightarrow SO(4)$, occurs at the scale f , giving rise to an $SU(2)_L$ doublet of NG bosons; at a lower scale $v = f \sin(\langle\pi\rangle/f) \equiv f \sin \theta$ the electroweak symmetry is spontaneously broken, $SO(4) \rightarrow SO(3)$, leaving an approximate custodial symmetry.

A simple way to derive the $SO(5)/SO(4)$ chiral lagrangian at $O(p^2)$ is by adopting the basis of fields $\{\chi^i, h\}$ and making use eq.(12). One has (see Appendix C):

$$\begin{aligned} \mathcal{L}^{(2)} &= \frac{f^2}{2} (D_\mu \Phi)^T (D^\mu \Phi) \\ &= \frac{1}{2} (\partial_\mu h)^2 + \frac{f^2}{4} \text{Tr}[(D_\mu \Sigma)^\dagger (D^\mu \Sigma)] \sin^2 \left(\theta + \frac{h(x)}{f} \right), \end{aligned} \quad (14)$$

where $\Sigma \equiv \exp(i\sigma^i \chi^i/v)$, and σ^i are the Pauli matrices. No covariant derivative acts on h , as one could have anticipated by noticing that the fluctuations parametrized by this field are $SO(4)'$ -invariant. Choosing the unitary gauge, $\Sigma = 1$, and expanding around θ , one immediately finds the relation $m_W^2 = (g^2 f^2 \sin^2 \theta)/4$, which determines the value of the electroweak scale $v = f \sin \theta$, and the value of the Higgs couplings to the vector bosons.

The same expression for $\mathcal{L}^{(2)}$ can be obtained by using the CCWZ formalism. At the level of two derivatives, there is only one operator which can be formed:

$$\mathcal{L}^{(2)} = \frac{f^2}{4} \text{Tr}[d_\mu d^\mu]. \quad (15)$$

The equivalence with eq.(14) is proved in Appendix C, but it can be quickly checked, for example, by monitoring the mass terms for the vector bosons. In the case of eq.(15) these arise from the component of the gauge fields along the broken generators contained in d_μ . From eq.(1) and (9), after setting $\Pi(x) = \sqrt{2} T^{\hat{a}}(\theta) \pi^{\hat{a}}(x)/f$, one finds:

$$d_\mu^{\hat{a}} = A_\mu^{\hat{a}} + \frac{\sqrt{2}}{f} (D_\mu \pi)^{\hat{a}} + O(\pi^3) \quad (16)$$

$$E_\mu^a = A_\mu^a - \frac{i}{f^2} (\pi \overleftrightarrow{D}_\mu \pi)^a + O(\pi^4). \quad (17)$$

For a generic θ , the components of the external gauge fields are given by

$$A_{\mu}^{\hat{a}} : \begin{cases} A_{\mu}^i = -\frac{\sin \theta}{\sqrt{2}} (W_{\mu}^i - \delta^{i3} B_{\mu}), & i = 1, 2, 3 \\ A_{\mu}^4 = 0 \end{cases} \quad (18)$$

$$A_{\mu}^a : \begin{cases} A_{\mu}^{a_L} = \left(\frac{1 + \cos \theta}{2} \right) W_{\mu}^a + \delta^{a3} \left(\frac{1 - \cos \theta}{2} \right) B_{\mu} \\ A_{\mu}^{a_R} = \left(\frac{1 - \cos \theta}{2} \right) W_{\mu}^a + \delta^{a3} \left(\frac{1 + \cos \theta}{2} \right) B_{\mu}, \end{cases} \quad (19)$$

where W_{μ}^a, B_{μ} are the $SU(2)_L \times U(1)_Y$ vector bosons. Using eq.(16) and (18) one can easily derive the W and Z mass terms from eq.(15).

Notice that in the case of $SO(5)/SO(4)$, the unbroken generators transform as a *reducible* representation of $SO(4) \sim SU(2)_L \times SU(2)_R$, namely they form a $(\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$. Each of the variables $E_{\mu}, f_{\mu\nu}^+$ can thus be divided into two individually covariant structures, aligned respectively along the $SU(2)_L$ and $SU(2)_R$ generators:

$$\begin{aligned} E_{\mu}^{L,R} &\equiv E_{\mu}^{a_{L,R}} T^{a_{L,R}}(\theta) & E_{\mu}^{L,R}(\Pi) &\rightarrow h(\Pi, g) E_{\mu}^{L,R}(\Pi) h^{\dagger}(\Pi, g) - i h(\Pi, g) \partial_{\mu} h^{\dagger}(\Pi, g) \\ f_{\mu\nu}^{L,R} &\equiv (f_{\mu\nu}^+)^{a_{L,R}} T^{a_{L,R}}(\theta) & f_{\mu\nu}^{L,R}(\Pi) &\rightarrow h(\Pi, g) f_{\mu\nu}^{L,R}(\Pi) h^{\dagger}(\Pi, g). \end{aligned} \quad (20)$$

In absence of the external gauging, the chiral lagrangian (15) has various discrete symmetries that lead to some important selection rules. Although a partial gauging of $SO(5)$ eventually breaks explicitly some of these parities, it is useful to identify them by thinking of $SO(5)$ as being fully gauged by external vectors, which is equivalent to formally assigning a transformation rule for $f_{\mu\nu}^{\pm}$. The following discrete symmetries, approximate or exact, are useful to describe the phenomenology of our model:

Grading of the algebra, R :

The quotient space $SO(5)/SO(4)$ is symmetric, that is, there exists an automorphism of the algebra (grading), R , under which the broken generators change sign:

$$\begin{aligned} T^{\hat{a}}(\theta) &\rightarrow -T^{\hat{a}}(\theta) \\ T^a(\theta) &\rightarrow +T^a(\theta) \end{aligned} \quad R = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -\cos(2\theta) & \sin(2\theta) \\ & & & \sin(2\theta) & \cos(2\theta) \end{pmatrix}. \quad (21)$$

The transformation rules of the fields are:

$$\begin{aligned} d_{\mu}^{\hat{a}} &\rightarrow -d_{\mu}^{\hat{a}} \\ \pi^{\hat{a}} &\rightarrow -\pi^{\hat{a}}, & E_{\mu}^a &\rightarrow +E_{\mu}^a \\ f_{\mu\nu}^{\pm} &\rightarrow \pm f_{\mu\nu}^{\pm}. \end{aligned} \quad (22)$$

Notice that for a generic θ , the action of R is linear on the $SO(5)/SO(4)$ NG bosons (all the π 's are odd), but non-linear on the fields χ^i, h . Since R is an element of the unbroken global $SO(4)$ (for any value of θ), i.e. it is an internal automorphism of the algebra, it will be an exact symmetry of the lagrangian at *any* chiral order in absence of the weak gauging. In particular, it will be unbroken to all orders in the chiral lagrangian. This implies that any process involving an odd number of NG bosons will vanish in the limit of zero electroweak couplings $g = g' = 0$. When the $SU(2)_L \times U(1)_Y$ gauging is turned on, the grading R is explicitly broken, except for $\theta = 0, \pi$.

LR parity, P_{LR} :

The parity P_{LR} exchanges the generators of the $SU(2)_L$ and $SU(2)_R$ subgroups of $SO(4)$, and changes sign to the first three broken generators ($\eta^{\hat{a}} \equiv (+1, +1, +1, -1)$):

$$\begin{aligned} T^{\hat{a}}(\theta) &\rightarrow -\eta^{\hat{a}} T^{\hat{a}}(\theta) \\ T^{a_L}(\theta) &\leftrightarrow T^{a_R}(\theta) \end{aligned} \quad P_{LR} = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & +1 & \\ & & & & +1 \end{pmatrix}. \quad (23)$$

This implies the following transformation rules for the NG bosons and the CCWZ covariant variables:

$$\begin{aligned} d_{\mu}^{\hat{a}} &\rightarrow -\eta^{\hat{a}} d_{\mu}^{\hat{a}} \\ E_{\mu}^L &\leftrightarrow E_{\mu}^R \\ f_{\mu\nu}^L &\leftrightarrow f_{\mu\nu}^R \\ (f_{\mu\nu}^-)^{\hat{a}} &\rightarrow -\eta^{\hat{a}} (f_{\mu\nu}^-)^{\hat{a}}. \end{aligned} \quad (24)$$

The action of P_{LR} is linear on the $SO(5)/SO(4)$ NG bosons, as well as on the χ^i, h fields, see eq.(13). In particular, one has: $\chi^i \rightarrow -\chi^i$ ($i = 1, 2, 3$), and $h \rightarrow +h$. The P_{LR} symmetry was invoked in the literature on composite Higgs models as a way to suppress large corrections to the $Zb\bar{b}$ vertex [14]. By eq.(24) we see that P_{LR} is an invariance of the Goldstone boson lagrangian at $O(p^2)$ (see also [15]), but, since it is not an element of $SO(4)$, one expects that it will generally be broken at $O(p^4)$, even in absence of the external gauging. In other words, P_{LR} is an accidental symmetry of the chiral lagrangian at $O(p^2)$. On the other hand, since $P_{LR} \in O(4)$, one could simply enforce it by requiring that the symmetry breaking pattern be $O(5) \rightarrow O(4)$. In either case, the accidental P_{LR} invariance at $O(p^2)$ implies that processes involving an odd number of χ 's, like the scattering of longitudinal vector bosons $W_L W_L \rightarrow Z_L h$, vanish at leading chiral order for $g, g' = 0$. In particular, this shows that the scattering amplitude for $W_L W_L \rightarrow Z_L h$ will behave like a constant at large energies $E \gg m_W, m_h$, rather than growing like E^2 . When the $SU(2)_L \times U(1)_Y$ gauging is turned on, P_{LR} is explicitly broken for any value of θ .

Higgs parity, P_h :

A further discrete symmetry can be constructed as the product of the grading R and P_{LR} : $P_h = R \cdot P_{LR}$. Like R , for generic θ , P_h is a symmetry at $O(p^2)$ in the gaugeless limit $g, g' = 0$. Its action on the NG bosons is such that the π^i are even ($i = 1, 2, 3$), while π^4 is odd: $\pi^i \rightarrow +\pi^i$, $\pi^4 \rightarrow -\pi^4$. In general, the action of P_h on the fields χ^i , h is non-linear, as can be deduced from their definition in terms of the π^a in eq.(13). However at linear order $h = \pi^4 + \dots$, so that π^4 interpolates an asymptotic state with one Higgs boson. In this sense, P_h acts like a ‘Higgs parity’ for any value of θ when neglecting g and g' . For the special value $\theta = \pi/2$, we have $P_h = P_5 \equiv \text{diag}(+1, +1, +1, +1, -1)$ which commutes with the gauged $SO(4)'$. In that case P_h is exact at $O(p^2)$ even in the presence of gauging, and it may as well be enforced as an exact symmetry to all chiral orders.⁵ Notice also that for $\theta = \pi/2$ the action becomes linear, $\chi^i \rightarrow +\chi^i$, $h \rightarrow -h$.

In addition to those described above, there are additional discrete symmetries which leave the lagrangian (15) invariant. Similarly to P_h , other three parities can be defined, P_i , under which the i -th NG boson changes sign while the others are even. Together with P_h , these parities correspond to the four Z_2 ’s contained in $O(4)$. Eq.(15) is also invariant under the spatial parity $P_0 : (t, \vec{x}) \rightarrow (t, -\vec{x})$, and the ordinary parity $P = P_0 \cdot P_{LR}$ under which the χ ’s transform as pseudo-scalars and the Higgs as a scalar. From the previous discussion it is clear that the ordinary parity P itself is accidental at order $O(p^2)$, and that it will be broken at higher orders. In the following we will not make any restrictive assumption of the symmetries possessed by the EWSB sector, and in particular we will allow for parity violation.

⁵If the couplings of the SM fermions to the strong sector preserve it, P_h can be enforced as an exact invariance of the full theory. This is possible for example in the MCHM4 [2] where fermions are embedded into spinorial representations of $SO(5)$, while in the MCHM5 [16], with fermions in fundamental representations of $SO(5)$, the fermion couplings break P_h . On the other hand, $v = f$ ($\theta = \pi/2$) is phenomenologically excluded in the MHCM5, as it implies vanishing fermion masses. An exact P_h invariance was invoked in Ref. [17] as a way to have a stable Higgs boson playing the role of a dark matter candidate.

2.2 The $SO(5)/SO(4)$ chiral lagrangian at $O(p^4)$ and its contribution to the scattering amplitudes

At the level of four derivatives, we find that a complete basis for the $SO(5)/SO(4)$ chiral lagrangian is made of 11 operators:

$$\begin{aligned}\mathcal{L}^{(4)} &= \sum_i c_i O_i \\ O_1 &= \text{Tr}[d_\mu d^\mu]^2 \\ O_2 &= \text{Tr}[d_\mu d_\nu] \text{Tr}[d^\mu d^\nu] \\ O_3 &= (\text{Tr}[E_{\mu\nu}^L E^{L\mu\nu}] - \text{Tr}[E_{\mu\nu}^R E^{R\mu\nu}]) \\ O_4^+ &= \text{Tr}[(f_{\mu\nu}^L + f_{\mu\nu}^R) i[d^\mu, d^\nu]] \\ O_5^+ &= \text{Tr}[(f_{\mu\nu}^-)^2] \\ O_4^- &= \text{Tr}[(f_{\mu\nu}^L - f_{\mu\nu}^R) i[d^\mu, d^\nu]] \\ O_5^- &= \text{Tr}[(f_{\mu\nu}^L)^2 - (f_{\mu\nu}^R)^2] \\ O_6^+ &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}[(f_{\mu\nu}^L + f_{\mu\nu}^R) i[d_\rho, d_\sigma]] \\ O_7^+ &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}[f_{\mu\nu}^- f_{\rho\sigma}^-] \\ O_6^- &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}[(f_{\mu\nu}^L - f_{\mu\nu}^R) i[d_\rho, d_\sigma]] \\ O_7^- &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}[f_{\mu\nu}^L f_{\rho\sigma}^L - f_{\mu\nu}^R f_{\rho\sigma}^R]\end{aligned}\tag{25}$$

Any other operator can be rewritten in terms of those shown above by means of identities valid in the case of a generic \mathcal{G}/\mathcal{H} symmetric space or specifically for the $SO(5)/SO(4)$ coset, see Appendices A and B.

It is useful to classify the O_i according to their behavior under P_{LR} and the ordinary parity P (the quantum numbers under P_h and P_0 can be easily derived in turn). One has that: O_1, O_2, O_4^+, O_5^+ are even under both P_{LR} and P ; O_3, O_4^-, O_5^- are odd under both P_{LR} and P ; O_6^+, O_7^+ are P_{LR} even and P odd; O_6^-, O_7^- are P_{LR} odd but P even.

We will be concerned in what follows with the size of the coefficients of the above operators. For that purpose it is worth reminding the reader that since those operators represent a subleading effect in the chiral lagrangian their coefficients evolve at 1-loop. In 1-loop accuracy we can thus conveniently write the value of the c_i at the weak scale as

$$c_i(m_Z) = c_i(\mu_{UV}) + \frac{b_i}{16\pi^2} \ln \frac{m_Z}{\mu_{UV}}\tag{26}$$

where μ_{UV} is some UV scale. The b_i are calculable $O(1)$ coefficient, fully determined by the $O(p^2)$ chiral lagrangian. In our discussion we shall mostly be concerned with physical situations where $c_i(\mu_{UV})$ dominate the above equation.

Only the first three operators in eq.(25) do not vanish in the limit in which the weak $SU(2)_L \times U(1)_Y$ gauging is turned off, and thus contribute at leading order to the scattering

of NG bosons. The other operators are thus not relevant for the following analysis and will not enter our discussion. The only exception is O_5^+ , which is the operator corresponding to the Peskin-Takeuchi S parameter [18]. It can be recast in the form (see Appendix C):

$$O_5^+ = \frac{1}{2} \sin^2 \left(\theta + \frac{h(x)}{f} \right) \left((W_{\mu\nu}^a)^2 + (B_{\mu\nu})^2 - \text{Tr} [\Sigma^\dagger W_{\mu\nu}^a \sigma^a \Sigma B_{\mu\nu} \sigma^3] \right), \quad (27)$$

where, we recall, $\Sigma = \exp(i\chi^i(x)\sigma^i/v)$. From the last term in the parenthesis it follows

$$\Delta\hat{S} = 2g^2 \sin^2 \theta c_5^+. \quad (28)$$

The logarithmically divergent contribution to this operator was discussed in Ref. [19], while Ref. [20] focussed on the UV-saturated contribution.

As a last remark we notice that the operator O_3 , being P_{LR} and P_h odd, would have the right quantum numbers to contribute to the scattering $W_L W_L \rightarrow Z_L h$, which does not arise at leading chiral order because it violates both P_{LR} and P_h . However, by switching off the external gauging and expanding O_3 one finds that the leading term with four NG bosons identically vanishes, that is: $O_3 \sim O(\pi^6)$. This is simply because, up to integration by parts, the only P_{LR} -odd structure one can form with four NG bosons and four derivatives is $\epsilon^{IJKL} \partial_\mu \pi^I \partial^\mu \pi^J \partial_\nu \pi^K \partial^\nu \pi^L$, which trivially vanishes by Bose symmetry. This implies that O_3 does not in fact contribute to $2 \rightarrow 2$ scattering amplitudes of NG bosons (although it will contribute to $2 \rightarrow 4$ ones). One can check that for $g = g' = 0$ the first non-vanish contribution to $W_L W_L \rightarrow Z_L h$ arises at $O(p^6)$, for instance from the operator $\epsilon^{IJKL} d_\mu^I (\nabla_\rho d^\mu)^J d_\nu^K (\nabla^\rho d^\nu)^L$.

Notice also in passing that for $SO(5)/SO(4)$ no Wess-Zumino-Witten term [21] can be constructed, since the fifth de Rham cohomology group of $SO(5)/SO(4)$ vanishes. Anyway such term would only affect amplitudes with at least five legs.

At this point we are ready to derive the contribution of the $O(p^4)$ lagrangian to the scattering amplitudes for $\pi\pi \rightarrow \pi\pi$. Since at linear order $\pi^i = \chi^i + \dots$ and $\pi^4 = h + \dots$ (see eq.(13)), by virtue of the Equivalence Theorem [22], the $\pi\pi \rightarrow \pi\pi$ amplitude reproduces the large-energy behavior of the amplitudes for the scattering among the Higgs and longitudinal vector bosons (see Appendix E). The general amplitude can be decomposed in terms of *two* functions:

$$\mathcal{A}(\pi^a \pi^b \rightarrow \pi^c \pi^d) = A(s, t, u) \delta^{ab} \delta^{cd} + A(t, s, u) \delta^{ac} \delta^{bd} + A(u, t, s) \delta^{ad} \delta^{bc} + B(s, t, u) \epsilon^{abcd}, \quad (29)$$

where s, t, u are the Mandelstam variables. The functions $A(s, t, u)$ and $B(s, t, u)$ are respectively associated with P_{LR} -even and P_{LR} -odd transitions. In particular, B parametrizes the amplitude for $W_L W_L \rightarrow Z_L h$ and crossed processes. Crossing symmetry requires that $A(s, t, u) = A(s, u, t)$, and that $B(s, t, u)$ be antisymmetric under the exchange of any two Mandelstam variables. The latter requirement implies that the lowest-order contribution to B arises at $O(p^6)$, that is $B \propto (s - u)(u - t)(t - s)$, in accordance with our previous discussion. From the $O(p^4)$ lagrangian given in eqs.(15) and (25), one can easily derive:

$$\begin{aligned} A(s, t, u) &= \frac{s}{f^2} + \frac{4}{f^4} [2c_1 s^2 + c_2 (t^2 + u^2)] \\ B(s, t, u) &= 0. \end{aligned} \quad (30)$$

Some comments concerning the sign and size of the coefficients $c_{1,2}$ are in order.⁶ Unitarity and causality are known to imply the constraint $c_1 + c_2 > 0$, $c_2 > 0$ [23]. However, that does not translate into a definite sign for the interference with the leading term when considering the squared amplitude. This can be seen by considering the plots of the following section where we consider the contributions to $c_{1,2}$ from heavy resonances, which satisfy the unitarity and causality bound. For instance, while the exchange of a vector enhances the W^+W^- final state and depletes hh , the converse is true for the exchange of a heavy singlet scalar. Let us consider instead the size of $c_{1,2}$. Notice that, by factoring out the leading term s/f^2 , the subleading corrections are characterized by the scales $f/\sqrt{c_{1,2}}$, that can be interpreted as the next physics threshold. If the only contribution to the chiral coefficients $c_{1,2}$ came from physics at the maximally allowed cutoff scale, $\Lambda_{\text{max}} \sim 4\pi f$, the one at which $\pi\pi$ scattering becomes maximally strong, then a naive estimate gives $c_{1,2} \sim 1/16\pi^2$. In that case the contribution of $O_{1,2}$ to the scattering amplitudes is always subleading for energies $E \lesssim 4\pi f$. If however some of the resonances of the strong sector where the global symmetry breaking takes place is somewhat lighter than the cutoff, then its contribution can become relevant. This possibility will be studied in the following sections.

3 Lagrangian for spin-1 and spin-0 resonances

We shall now consider the case in which one single resonance is lighter than the cutoff scale.

Before going into the details we should briefly illustrate our assumptions. By necessity, the cut-off Λ of our effective σ -model description should satisfy $\Lambda \leq 4\pi f$, since the latter is the scale where it becomes strongly coupled. We could also trade Λ for the σ -model coupling at that scale $g_* \equiv \Lambda/f$. At around the scale Λ we expect new states, and if the coupling strength is universal, they will couple to each other with strength g_* . Our assumption now is that there is one further resonance Φ with mass $m_\Phi < \Lambda$, whose interactions to the NG bosons and to itself are described by a set of couplings $\{g_\Phi^i\}$. We can think of two plausible criteria to constrain the couplings $\{g_\Phi^i\}$ and to assess their most likely range. The first and less conservative criterion is given by the request that the new couplings stay weak up to the cut-off scale Λ . The second, similar but stronger, is that the couplings should not exceed, and preferably saturate, the σ -model coupling $g_* \equiv \Lambda/f$ at the scale Λ . This second criterion is perhaps more plausible than the first one, as it is basically realized at large N or in weakly-coupled 5D field theories, with g_* equalling respectively $4\pi/\sqrt{N}$ and the Kaluza-Klein coupling g_{KK} . Moreover, as will become clear below, one interesting consequence of couplings that saturate the second criterion is that at $s > m_\Phi^2$ the leading linear growth of $\mathcal{A}(\pi\pi \rightarrow \pi\pi)$ with s changes by $O(1)$ with respect to the pure σ -model result at $s \ll m_\Phi^2$. This means that Φ is as important as the heavier resonances in the UV completion of $\pi\pi$ scattering, or, in other words, that its exchange *partially UV completes* $\pi\pi$ scattering. In view of that implication we shall from now on refer to the second (stronger) criterion as Partial UV Completion (PUVC). Notice that the request that the new resonance Φ “unitarizes” or minimizes the amplitude, as for instance requested in Ref. [8], is a very special case of partial UV completion, in particular it leads to couplings of similar size. PUVC, as we defined it, is however distinguished from “unitarization”, as it does not necessarily imply a

⁶We assume $c_{1,2}(\mu_{UV})$ dominates over the 1-loop running contribution to $c_{1,2}$.

reduced scattering amplitude, or if so only a partial reduction. Keeping in mind our goal of parametrizing unknown physics in as much generality as possible, PUVc seems a reasonable criterion to adopt.⁷ In view of its more interesting properties, and in view of the parameter reduction it entails, we will adopt PUVc in our phenomenological analysis in section 4.

We will study only the case of resonances which have the right quantum numbers under $SO(4)$ to be exchanged in $\pi\pi$ scattering. Since the NG bosons transform as a **4** of $SO(4)$, the resonance must fall in one of the following representations: $\mathbf{4} \times \mathbf{4} = \mathbf{1} + \mathbf{6} + \mathbf{9}$ of $SO(4)$, that is $(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{3})$ of $SU(2) \times SU(2)$. Bose symmetry then implies that the possible spin assignments are (we consider only spin 0 and spin 1 resonances):

- $\eta = (\mathbf{1}, \mathbf{1})$, spin = 0 [isospin = 0]
- $\rho^L = (\mathbf{3}, \mathbf{1})$, $\rho^R = (\mathbf{1}, \mathbf{3})$, spin = 1 [isospin = 1]
- $\Delta = (\mathbf{3}, \mathbf{3})$, spin = 0 [isospin = 0 + 1 + 2]

In parenthesis we have indicated the decomposition under $SO(3)$ (isospin) quantum numbers; for example, the scalar Δ contains three components respectively with isospin 0, 1, 2.

In the following we will consider each of above resonances, writing down its chiral lagrangian and computing its contribution to the $\pi\pi \rightarrow \pi\pi$ scattering amplitude. Let us start with the case of a spin-1 ρ^L .

3.1 $\rho^L = (3, 1)$

We follow the vector formalism and describe the spin-1 resonance with a field ρ_μ transforming non-homogeneously,⁸

$$\rho_\mu \rightarrow h(\Pi, g) \rho_\mu h^\dagger(\Pi, g) - i h(\Pi, g) \partial_\mu h^\dagger(\Pi, g), \quad (31)$$

so that a field strength can be defined as $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + i[\rho_\mu, \rho_\nu]$. We are interested in a physical situation where the interactions among the NG bosons and the ρ at a scale of the order of its mass m_ρ are weak and described by the lowest terms in a derivative expansion. For instance, a plausible assumption is that the derivative expansion is controlled by ∂/Λ , the cut-off scale satisfying $m_\rho \ll \Lambda \leq 4\pi f$. It is thus convenient to write the effective lagrangian for the ρ in the form

$$\mathcal{L}^{(\rho_L)} = -\frac{1}{4g_{\rho_L}^2} \rho_{\mu\nu}^{a_L} \rho^{a_L \mu\nu} + \frac{m_{\rho_L}^2}{2g_{\rho_L}^2} (\rho_\mu^{a_L} - E_\mu^{a_L})^2 + \sum_i \alpha_i Q_i, \quad (32)$$

where Q_i denote the (infinite) series of higher-order (in field and derivative) operators. Without these latter, $\mathcal{L}^{(\rho_L)}$ coincides with the lagrangian originally considered in $SO(4)/SO(3)$ models of Hidden Local Symmetry (HLS) [25, 26, 27]. For phenomenological applications we are interested in operators that affect $\pi\pi$ scattering as well as operators that play a role in precision electroweak tests by modifying the electroweak vector boson propagators. Within

⁷It must however be noted that Ref. [8] emphasizes how in low-energy QCD the ρ vector meson seems to remarkably closely adhere to the more restricted request of “unitarization”.

⁸See for example Ref. [24] for a comparison among different formalisms.

this limited but reasonable perspective, a straightforward operator analysis singles out four leading independent operators

$$\begin{aligned}
Q_1 &= \text{Tr}(\rho^{\mu\nu} i[d_\mu, d_\nu]) \\
Q_2 &= \text{Tr}(\rho^{\mu\nu} f_{\mu\nu}^+) \\
Q_3 &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}(\rho_{\mu\nu} i[d_\rho, d_\sigma]) \\
Q_4 &= \epsilon^{\mu\nu\rho\sigma} \text{Tr}(\rho_{\mu\nu} f_{\rho\sigma}^+) .
\end{aligned} \tag{33}$$

In Appendix F we give more details on the analysis that leads to the above result and also list additional operators that can affect other processes, like for example $\rho\pi \rightarrow \rho\pi$. Operators that do not involve the $\rho_{\mu\nu}$ field strength are written in terms of $\bar{\rho}_\mu \equiv \rho_\mu - E_\mu$. For instance, in order to illustrate our working assumption, it is useful to consider the operator $Q_{1(2)} = (\nabla^\mu \bar{\rho}_\mu)^2$, which we did not list among the leading ones. This operator has the same structure as the ρ mass term, but with two additional derivatives. According to our assumption that the derivative expansion is controlled by the scale $\Lambda = g_* f \gg m_\rho$, we must have $\alpha_{1(2)} \lesssim 1/g_*^2$. This is sensible since, as it is well known, $Q_{1(2)}$ implies the presence of a scalar ghost with mass $m_\rho/(g_\rho \sqrt{\alpha_{1(2)}})$.

Among the leading operators, Q_1 and Q_3 in principle affect the $\pi\pi\rho$ vertex, and thus $\pi\pi$ scattering, but in fact only Q_1 does, since the contribution from Q_3 is easily shown to vanish because of Bose symmetry. The mass term also includes a contribution to the $\pi\pi\rho$ vertex, as well as a contact interaction among four NG bosons. Upon expanding in the number of NG fields and using the commutation relations of eq.(89), one has $(i, j, k = 1, 2, 3, a, b = 1, 2, 3, 4)$:

$$\begin{aligned}
\mathcal{L}^{(\rho_L)} \supset & \frac{a_{\rho_L}^2 g_{\rho_L}}{2} [\epsilon^{ijk} \pi^i \partial^\mu \pi^j \rho_\mu^k + (\pi^k \partial^\mu \pi^4 - \pi^4 \partial^\mu \pi^k) \rho_\mu^k] \\
& - \frac{a_{\rho_L}^2}{16f^2} [(\pi^a \partial_\mu \pi^a)^2 - (\pi^a \partial_\mu \pi^b)^2] \\
& - \frac{2g_\rho \alpha_1}{f^2} [\epsilon^{ijk} \partial^\mu \pi^i \partial^\nu \pi^j \partial_\mu \rho_\nu^k - (\partial^\mu \pi^4 \partial^\nu \pi^k - \partial^\mu \pi^k \partial^\nu \pi^4) \partial_\mu \rho_\nu^k] + \dots ,
\end{aligned} \tag{34}$$

where we have rescaled the ρ field in order to canonically normalize its kinetic term and we have defined

$$a_{\rho_L} \equiv \frac{m_{\rho_L}}{g_{\rho_L} f} . \tag{35}$$

At energies $E \ll m_{\rho_L} \ll \Lambda$ one can integrate out the ρ by solving the equations of motion at lowest order in the derivative expansion. One has ⁹

$$\rho_\mu^L = E_\mu^L + O\left(\frac{E^2}{m_{\rho_L}^2}\right) . \tag{36}$$

This implies, first of all, that the $O(p^2)$ lagrangian in eq.(15) is not renormalized by the exchange of the ρ [28]. As concerns the $O(p^4)$ lagrangian in eq.(25), we notice that Q_5 and

⁹Notice that in the power counting of the low-energy effective lagrangian ρ_μ counts as one derivative.

Q_6 discussed in the Appendix, as well as all the operators with two or more $\bar{\rho}'s$, vanish at leading chiral order on the equations of motions, since they contain at least one power of $\bar{\rho}$. Those operators therefore do not affect the $O(p^4)$ low-energy lagrangian. The latter is only affected by Q_1 - Q_4 . Plugging (36) into (32) gives rise to the effective lagrangian of eq.(25) with coefficients

$$\begin{aligned}
c_1 &= -c_2 = \frac{1}{2}c_3 = -\frac{1}{4}\left(\alpha_1 + \frac{1}{4g_{\rho_L}^2}\right) \\
c_4^+ &= \frac{1}{2}(\alpha_1 - \alpha_2) + \frac{1}{4g_{\rho_L}^2} \\
c_5^+ &= \frac{1}{2}\left(\frac{1}{4g_{\rho_L}^2} - \alpha_2\right) \\
c_4^- &= -c_5^- = -\frac{1}{2}(\alpha_2 + \alpha_1) \\
c_6^+ &= \frac{1}{2}(\alpha_3 - \alpha_4) \\
c_7^+ &= -\frac{\alpha_4}{2} \\
c_6^- &= -c_7^- = -\frac{1}{2}(\alpha_3 + \alpha_4) .
\end{aligned} \tag{37}$$

Similarly, it can be shown that by integrating out a $\rho^R = (\mathbf{1}, \mathbf{3})$, one obtains the same value for all the coefficients of the P_{LR} -even operators ($c_1, c_2, c_4^+, c_5^+, c_6^+, c_7^+$), and the same value but opposite sign for the the coefficients of the P_{LR} -odd operators ($c_3, c_4^-, c_5^-, c_6^-, c_7^-$).

By the above equations and from the results of the previous section, we can read the effects of the various parameters on low-energy quantities: α_2 affects the S parameter while α_1 and g_ρ determine the $O(s^2)$ corrections to the $\pi\pi$ scattering amplitude. More explicitly we have

$$A(s, t, u) = \frac{s}{f^2} + \frac{1}{f^4}\left(\alpha_1 - \frac{1}{4g_{\rho_L}^2}\right)(2s^2 - t^2 - u^2) \tag{38}$$

$$B(s, t, u) = 0$$

and

$$\Delta\hat{S} = a_{\rho_L}^2 \frac{m_W^2}{m_{\rho_L}^2} - 4\alpha_2 \frac{m_W^2}{v^2} \xi . \tag{39}$$

The amplitude at energies of order m_ρ depends on just one additional parameter, m_ρ itself, and we shall present it later.

In order to assess which are the most plausible regions of parameter space, we should study the theoretical constraints on the above parameters. Those are mostly associated to the strength of the interactions at energies around or above m_ρ . In the latter regime the longitudinal and transverse polarizations of the ρ behave differently. The power counting is more readily done by parametrizing the longitudinal polarizations, ρ_L , in terms of an additional set of eaten NG bosons. The convenient way to proceed is to first formally

take the limit $g_\rho \rightarrow 0$ with m_ρ/g_ρ fixed, characterize the σ -model so obtained, and then re-introduce the transverse ρ_T field with coupling g_ρ by gauging a subgroup of the global symmetry. It is rather clear that the σ -model one obtains in the first step can be written as $SO(5) \times SU(2)_H/SU(2)'_L \times SU(2)_R$, where, in an obvious notation, the embedding is

$$SU(2)_L \times SU(2)_R \sim SO(4) \subset SO(5) \quad SU(2)'_L = [SU(2)_L \times SU(2)_H]_{\text{diag}}. \quad (40)$$

This coset is parametrized by a set of 7 Goldstone bosons transforming as $(2, 2) \oplus (3, 1)$ under $SU(2)'_L \times SU(2)_R$, which we indicate as $\pi^{\hat{i}} = (2, 2)$ ($\hat{i} = 1, \dots, 4$) and $\eta^a = (3, 1)$ ($a = 1, 2, 3$). As explained in Appendix D, a suitable parametrization of the coset when performing the CCWZ construction is $U = e^{i\pi} e^{i\eta}$. This way the d symbols depend on π and η according to

$$(2, 2) \rightarrow d_\mu^{\hat{i}} \equiv d_\mu^{\hat{i}}(\pi) \sim \partial_\mu \pi + \pi^2 \partial_\mu \pi + \dots \quad (41)$$

$$(3, 1) \rightarrow \tilde{d}_\mu^a \equiv \tilde{d}_\mu^a(\pi, \eta) \sim \partial_\mu \eta + \pi \partial_\mu \pi + \eta^2 \partial_\mu \eta + \dots \quad (42)$$

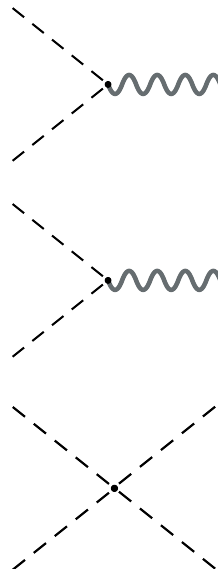
Notice that in \tilde{d} there is an $O(\pi^2)$ term corresponding to the fact that the coset is not a symmetric space. Finally, since the coset is reducible, the $O(p^2)$ lagrangian is determined by two independent parameters (decay constants):

$$\mathcal{L} = f^2 d_\mu d^\mu + f_\rho^2 \tilde{d}_\mu \tilde{d}^\mu. \quad (43)$$

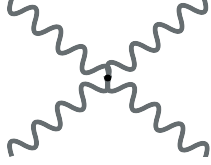
By gauging $SU(2)_H$ by the introduction of ρ_μ , the second term in the above equation is mapped into the ρ mass term, hence the identification

$$m_\rho = g_\rho f_\rho \rightarrow a_\rho = \frac{f_\rho}{f}. \quad (44)$$

We have now all the ingredients to estimate the strength of the interactions among $(W_L, \rho_L, \rho_T) \sim (\pi, \eta, \rho_T)$. Focussing on the ρ kinetic lagrangian and on Q_1 , we find the following result:



$$\begin{aligned} \mathcal{A}(\pi\pi\rho_T) &\sim g_\rho \left[\frac{f_\rho^2}{f^2} + \alpha_1 \frac{E^2}{f^2} \right] \\ \mathcal{A}(\pi\pi\rho_L) &\sim \frac{f_\rho E}{f^2} \\ \mathcal{A}(\pi^4) &\sim \frac{f_\rho^2 E^2}{f^4} + \frac{E^2}{f^2} \end{aligned}$$



$$\mathcal{A}(\rho_L^4) \sim \frac{E^2}{f_\rho^2}$$

Assuming now the cut-off scale is $\Lambda \equiv g_* f \ll 4\pi f$, we can apply the two criteria discussed at the beginning of this section. The less conservative criterion corresponds to the request that the above interactions stay weak below Λ . This leads to the constraints

$$\frac{\Lambda}{4\pi f} \lesssim a_\rho \lesssim \frac{4\pi f}{\Lambda}, \quad \alpha_1 < \frac{1}{g_\rho g_*} \frac{4\pi}{g_*}. \quad (45)$$

We thus conclude that a_ρ is allowed to vary in a range around 1. The coefficient α_1 is bounded from above, but as long as $g_*^2 < 4\pi g_\rho$, it can be as large as $1/g_\rho^2$, in which case it contributes significantly to $\pi\pi$ scattering also at and below the ρ threshold (see equations in footnote 12).

If we make the stronger, and maybe more realistic request that all couplings do not exceed g_* at $g_* f$,¹⁰ we simply find

$$a_\rho \sim 1, \quad \alpha_1 < \frac{1}{g_\rho g_*} < \frac{1}{g_\rho^2}. \quad (46)$$

where in the last inequality we used that $g_\rho < g_*$, which follows from the hypothesis $m_\rho < g_* f$ for $a_\rho \sim 1$. Using the above relations and eqs.(52), (53) one can check that the contribution of the ρ partially UV completes $\mathcal{A}(\pi\pi \rightarrow \pi\pi)$, i.e. it changes by $O(1)$ the rate of growth of the leading term (see also the estimate of the π^4 contact interaction in the figure above). That motivates us to simply call the second (stronger) criterion Partial UV Completion. Notice indeed that in this case the ρ behaves approximately like an elementary gauge boson, with the form factor parameter α_1 negligible around the ρ peak (see the $\pi\pi\rho$ amplitude estimates in the figure).

One final constraint concerns the parameter α_2 . In principle the operator Q_2 belongs to a different class compared to Q_1 , as it involves a coupling between the ρ and the elementary gauge fields. In this sense it might not be subject to the request of having a strength $< g_*$ at the scale Λ , as imposed by PUV. There is however a constraint that α_2 must satisfy for consistency: this parameter contributes an off-diagonal kinetic term for the vectors, so that when it is sufficiently large one vector will turn into a ghost. In order to avoid that, a bound must be satisfied:

$$\alpha_2 < \frac{1}{g_\rho g_{SM}}. \quad (47)$$

This bound leaves plenty of space for α_2 to give a sizeable contribution to S . In particular, in the range $\alpha_2 \sim 1/g_\rho^2$ and for $\alpha_2 > 0$, the contribution to S can compensate the positive one from ρ exchange (see eq.(39)). In our study, we shall also consider the possibility to have rather low values of the ρ mass, $m_\rho \sim 1$ TeV, in a region where it is disfavored by the bounds on S . It is thus reassuring that we can compensate for S with a slight tuning of α_2 . It is also

¹⁰More precisely, we require the trilinear couplings to be $< g_*$ and the quadrilinears to be $< g_*^2$.

interesting to notice that a value $\alpha_2 \sim 1/g_\rho^2$ can lead to Vector Meson Dominance (VMD) in the form factor of the pion to the external $SU(2)_L \times U(1)_Y$ gauge fields (the analog of the electromagnetic form factor of the pion in QCD) for a generic $a_\rho \sim O(1)$.¹¹

While $\alpha_2 \sim 1/g_\rho^2$ is possible under the assumption of PUVC and also compatible with the request of Vector Meson Dominance in the $SU(2)_L$ form factor of the NG bosons, perhaps a more reasonable estimate of α_2 from physics at the scale $g_* f$ is precisely $\alpha_2 \sim 1/g_*^2$, in which case it would be subleading. One should also recall that in explicit models based on 5D theories or deconstruction, where ρ is followed by a tower of vectors, the total contribution to α_2 and c_5^+ from the heavy resonances always corresponds to a positive S . But within our limited perspective we can only parameterize S , rather than calculate it.

It is useful to compare with the approach adopted by the authors of Ref. [9], who studied the effect of the exchange of a vector resonance V in the context of an $SO(4)/SO(3)$ theory. They require, as a rationale for estimating the coefficients of the operators in the chiral lagrangian of the resonance, that the scattering amplitudes for $\pi\pi \rightarrow VV$ do not grow faster than E^2 , and that those for $\psi\bar{\psi} \rightarrow VV$ (where ψ is a SM fermion) be at most constant at large energies. In our notation that corresponds to requiring negligible coefficients for all the Q_i , which is also true in PUVC. However PUVC is more restrictive as it further constrains the coefficients of the amplitude at $O(E^2)$.

Having discussed in detail our assumptions and their implications on the structure of the lagrangian, we now turn to the study of the contribution of the ρ to $\pi\pi$ scattering. We do so by assuming PUVC and thus neglecting the contribution of the operators Q_i . Using eq.(34), it is straightforward to derive the expression of the scattering amplitudes valid up

¹¹For example, by making use of the lagrangian $\mathcal{L}^{(\rho_L)}$ and keeping only the ρ kinetic and mass terms and Q_2 , we find that for $\theta = 0$ the form factor that parametrizes the interaction of the pion with the $SU(2)_L$ gauge field is given by

$$F(q^2) = 1 - a_\rho^2 - \frac{a_\rho^2 m_\rho^2}{q^2 - m_\rho^2} \left(1 - 2g_\rho^2 \alpha_2 \frac{q^2}{m_\rho^2} \right). \quad (48)$$

The third term is due to the exchange of the ρ , while the second follows from the direct $W\pi\pi$ interaction that comes from the ρ mass term. The request of Vector Meson Dominance, namely that $F(-\infty) = 0$, is satisfied for

$$\alpha_2 = \frac{a_\rho^2 - 1}{2a_\rho^2} \frac{1}{g_\rho^2}. \quad (49)$$

It thus follows that for $a_\rho > 1$ (so that $\alpha_2 > 0$) a reduction in S is possible. More precisely, by substituting eq.(49) in eq.(39) one obtains $\Delta\hat{S} = (2 - a_\rho^2)m_W^2/m_\rho^2$. An exact cancellation of S then occurs for $a_\rho^2 = 2$. Notice that for $a_\rho^2 = 2$ one also obtains the two relations

$$g_{\rho\pi\pi} = g_\rho \quad (50)$$

$$m_\rho^2 = 2g_{\rho\pi\pi}^2 f^2, \quad (51)$$

where $g_{\rho\pi\pi}$ denotes the coupling of the ρ to the NG bosons. The analog relations in QCD go respectively under the name of ‘coupling universality’ and (a possible formulation of) the KSFR relation (see for example Ref. [25]). In general, from eq.(34) and (35) it follows (after neglecting the subleading contribution to $g_{\rho\pi\pi}$ from α_1): $g_{\rho\pi\pi} = a_\rho^2 g_\rho / 2$ and $m_\rho^2 = 4g_{\rho\pi\pi}^2 f^2 / a_\rho^2$. Notice also that for a vanishing α_2 , the request of VMD can be satisfied for $a_\rho^2 = 1$, for which the relations (50), (51) do not hold. This is in contrast with the case of QCD, where VMD, universality of couplings and the KSFR relation can all be obtained from a lagrangian containing just the kinetic and mass term of the ρ .

to energies $E \ll \Lambda$. We find:¹²

$$\begin{aligned}
A(s, t, u) &= \frac{s}{f^2} \left(1 - \frac{3}{4} a_{\rho_L}^2 \right) - \frac{1}{4} a_{\rho_L}^2 \frac{m_{\rho_L}^2}{f^2} \times \\
&\quad \times \left[\frac{s - u}{t - m_{\rho_L}^2 + i\Gamma_{\rho_L} m_{\rho_L} \theta(t)} + \frac{s - t}{u - m_{\rho_L}^2 + i\Gamma_{\rho_L} m_{\rho_L} \theta(u)} \right] \\
B(s, t, u) &= \frac{1}{4} a_{\rho_L}^2 \frac{m_{\rho_L}^2}{f^2} \left[\frac{u - t}{s - m_{\rho_L}^2 + i\Gamma_{\rho_L} m_{\rho_L} \theta(s)} + \frac{s - u}{t - m_{\rho_L}^2 + i\Gamma_{\rho_L} m_{\rho_L} \theta(t)} \right. \\
&\quad \left. + \frac{t - s}{u - m_{\rho_L}^2 + i\Gamma_{\rho_L} m_{\rho_L} \theta(u)} \right],
\end{aligned} \tag{53}$$

where we have included the imaginary part in the denominators to take into account the finite width, Γ_{ρ_L} , of the ρ^L . As expected $B(s, t, u) \sim (E/m_{\rho_L})^6$ at energies $E \ll m_{\rho_L}$. The expression of $A(s, t, u)$ coincides with that previously obtained in the literature for a vector of $SO(3)$ in $SO(4)/SO(3)$ models, see for example Ref.[6].¹³ As for the $SO(4)/SO(3)$ case [8], we also find that the $\pi\pi$ elastic scattering can be “perturbatively unitarized” for $a_{\rho_L} = 2/\sqrt{3}$ (for the same value, however, the inelastic channels remain strongly coupled). In the case of a $\rho^R = (\mathbf{1}, \mathbf{3})$, the $\pi^4 \pi^i \rho^i$ interaction (the second term in square brackets in the second line of eq.(34)) changes sign, which implies that the amplitude $B(s, t, u)$ also changes sign, while $A(s, t, u)$ is the same as for a ρ^L .

Figure 2 reports the cross section of $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, $W_L^+ W_L^- \rightarrow hh$ and $W_L^+ W_L^- \rightarrow Z_L h$, where all the gauge bosons are assumed to be longitudinally polarized (see Appendix E for the expressions of the scattering amplitudes among longitudinal vector bosons). The following values of input parameters have been chosen: $\xi \equiv (v/f)^2 = \sin^2 \theta = 0.5$, $m_{\rho_L} = 1.5 \text{ TeV}$ and $a_{\rho_L} = 2/\sqrt{3}$. The ρ^L decay constant has been computed including only the decays to the NG bosons. We find

$$\Gamma_{\rho_L} = \frac{a_{\rho_L}^2 m_{\rho_L}^3}{96\pi f^2}, \tag{54}$$

which gives $\Gamma_{\rho_L} = 123 \text{ GeV}$ for the above set of numerical inputs. In Fig. 2 we have not

¹² For completeness we report the contribution to the $\pi\pi \rightarrow \pi\pi$ amplitudes generated by an insertion of the operator Q_1 . This is the only additional operator contributing to the scattering amplitude that is not negligible under the less stringent assumption of perturbativity up to a scale $g_* f > m_\rho$. We find ($\tilde{\alpha}_1 \equiv \alpha_1 g_{\rho_L}^2$)

$$\begin{aligned}
A(s, t, u) &= \frac{a_{\rho_L}^2 \tilde{\alpha}_1}{f^2} \left[\frac{u^2 - s^2}{t - m_{\rho_L}^2} + \frac{t^2 - s^2}{u - m_{\rho_L}^2} \right] + \frac{a_{\rho_L}^2 \tilde{\alpha}_1^2}{f^2} \left[\frac{t^2(u - s)}{m_{\rho_L}^2(t - m_{\rho_L}^2)} + \frac{u^2(t - s)}{m_{\rho_L}^2(t - m_{\rho_L}^2)} \right], \\
B(s, t, u) &= -a_{\rho_L}^2 \tilde{\alpha}_1 (1 - \tilde{\alpha}_1) \frac{m_{\rho_L}^2}{f^2} \left[\frac{(t - u)(u - s)(s - t)}{(s - m_{\rho_L}^2)(t - m_{\rho_L}^2)(u - m_{\rho_L}^2)} \right].
\end{aligned} \tag{52}$$

In order to get a more compact expression, we have dropped the decay widths in the propagators, as they can be reintroduced straightforwardly. Notice that B is completely antisymmetric in the Mandelstam variables as required by Bose symmetry.

¹³A meaningful comparison requires to define the parameters g_ρ and f in both theories by means of physical observables. In particular, g_ρ can be defined as the strength of the interaction among three ρ 's.

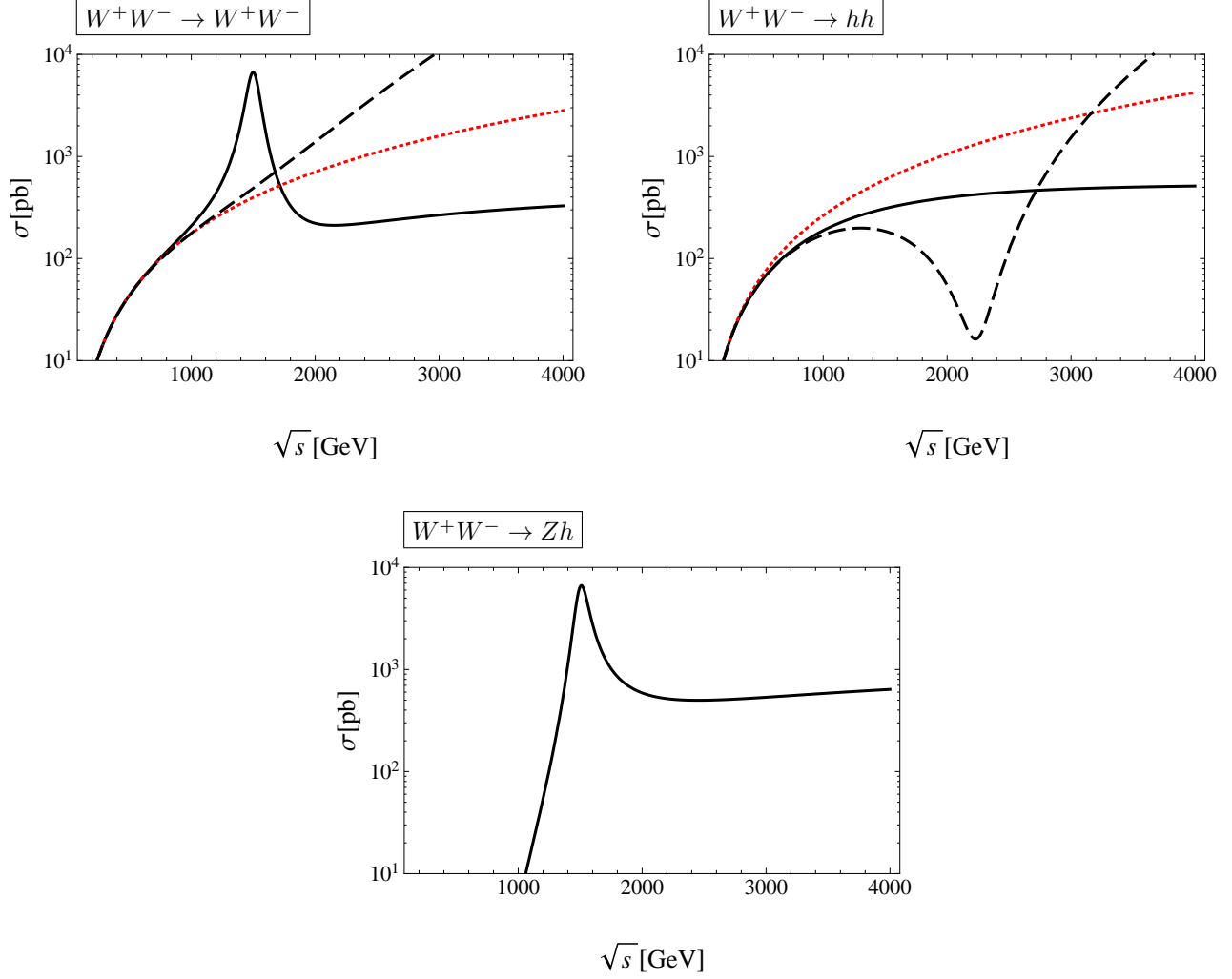


Figure 2: Contribution of ρ^L to the $W_L^+W_L^- \rightarrow W_L^+W_L^-$ (upper left), $W_L^+W_L^- \rightarrow hh$ (upper right) and $W_L^+W_L^- \rightarrow Z_L h$ (lower panel) cross sections for $\xi = 0.5$, $m_{\rho_L} = 1.5$ TeV and $a_{\rho_L} = 2/\sqrt{3}$, which implies $\Gamma_{\rho_L} = 123$ GeV. The dotted red and dashed black curves respectively show the $O(p^2)$ and $O(p^4)$ predictions, as obtained by using eq.(38) with $\alpha_1 = 0$. The solid black curve shows the full effect of the ρ^L exchange, as computed by means of eq.(53).

shown the cross sections of $W_L^\pm Z_L \rightarrow W_L^\pm Z_L$ and $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ because they are qualitatively equal to those of $W_L^+W_L^- \rightarrow W_L^+W_L^-$ and $W_L^+W_L^- \rightarrow hh$ respectively.

As one could have anticipated (see for example Ref. [7]), the contribution of the ρ enhances those processes where it can be exchanged in s channel, while it suppresses those where it only enters via the t and u channels. More specifically, $W_L^+W_L^- \rightarrow W_L^+W_L^-$ and $W_L^\pm Z_L \rightarrow W_L^\pm Z_L$ are enhanced, while $W_L^+W_L^- \rightarrow hh$ and $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ get suppressed compared to the $O(p^2)$ result. Notice that the t - and u -channel contributions of ρ^L to $W_L W_L \rightarrow hh$ follow from the existence of the coupling $\rho^L \chi h$. In the case of $SO(4)/SO(3)$, with h an ordinary neutral scalar, that coupling can be forbidden by imposing parity invariance. That is why the authors of Ref. [10] find no contribution to $W_L W_L \rightarrow hh$ from the exchange of a vector resonance.

3.2 $\eta = (\mathbf{1}, \mathbf{1})$

We now consider the case of a scalar resonance η transforming like a $(\mathbf{1}, \mathbf{1})$ of $SU(2) \times SU(2)$ (a singlet of $SO(4)$). As done for the ρ , we write the effective lagrangian by focussing on the leading operators in a derivative expansion ∂/Λ that are relevant for $\pi\pi$ scattering. We find the lagrangian

$$\mathcal{L}^{(\eta)} = \frac{1}{2} (\partial_\mu \eta)^2 - \frac{1}{2} m_\eta^2 \eta^2 + \frac{f^2}{4} \left(2a_\eta \frac{\eta}{f} + b_\eta \frac{\eta^2}{f^2} \right) \text{Tr} [d_\mu d^\mu] , \quad (55)$$

where cubic and quartic self-interactions for η have been omitted because not relevant for the following. It contains the following $\eta\pi\pi$ interaction term:

$$\mathcal{L}^{(\eta)} \supset a_\eta \frac{\eta}{f} \partial_\mu \pi^a \partial^\mu \pi^a + \dots \quad (56)$$

We can now bound the size of the couplings under the same assumptions of the previous section. The PUVc request that no coupling exceeds g_* at the putative cut-off scale $g_* f$ implies

$$a_\eta, b_\eta \lesssim O(1) . \quad (57)$$

More specifically, for the range $a_\eta = O(1)$ the exchange of η does partially UV complete $\pi\pi$ scattering as shown in eq.(62) below. Indeed, for $a_\eta = b_\eta = 1$ the lagrangian (55) describes a linear sigma model where the scalar η and the $SO(5)/SO(4)$ NG bosons fit together in a fundamental (linearly-transforming) representation of $SO(5)$. For that particular choice all the scattering amplitudes are perturbatively unitarized provided η is lighter than the cutoff [19]. On the other hand, by making the more conservative request that couplings be just perturbative at $g_* f$ we would obtain

$$a_\eta^2, b_\eta \lesssim O(16\pi^2/g_*^2) , \quad (58)$$

by which the contribution from a_η could well dominate $\pi\pi$ scattering around the η peak.

It may be instructive to contemplate the possible effects of higher derivative terms, not displayed in eq.(55). For instance the term $\eta \partial^2 (d_\mu d^\mu)$, involves two extra powers of p with respect to the term proportional to a_η . Its role is thus analogous to that of Q_1 in the $\rho\pi\pi$ vertex. Of course under the request that all couplings do not exceed g_* at $g_* f$ this term, as all higher derivative terms, is subleading. By making the weaker request that the couplings be just perturbative at $g_* f$ one finds that the new operator can affect at $O(1)$ the amplitude around $\sqrt{s} \sim m_\eta$ only if $g_*^3 < 4\pi(m_\eta/f)^2$. The latter constraint implies very little separation between m_η and the cutoff $g_* f$. In view of the above, in the following we will neglect higher-derivative terms.

Notice that the lagrangian (55) is obviously invariant under P_{LR} , by assigning η positive P_{LR} parity. This means that the exchange of η will not mediate any P_{LR} -violating process, like $W_L W_L \rightarrow Z_L h$, nor will it generate P_{LR} -odd operators once integrated out at low energy.

At energies $E \ll m_\eta$ the field η can be integrated out by solving the equations of motion at leading order. This gives

$$\eta = \frac{a_\eta f}{2m_\eta^2} \text{Tr} [d_\mu d^\mu] \left(1 + O \left(\frac{E^2}{m_\eta^2} \right) \right) . \quad (59)$$

By plugging the above solution into eq.(55), one obtains a low-energy chiral lagrangian of the form (25) with

$$c_1 = \frac{a_\eta^2 f^2}{8m_\eta^2} \quad (60)$$

and all the remaining chiral coefficients equal to zero. From eq.(30) then it follows that, for $E \ll m_\eta$,

$$\begin{aligned} A(s, t, u) &= \frac{s}{f^2} + a_\eta^2 \frac{s^2}{m_\eta^2 f^2} \\ B(s, t, u) &= 0. \end{aligned} \quad (61)$$

Full inclusion of the contribution from the η exchange up to energies $E \ll \Lambda$ gives

$$\begin{aligned} A(s, t, u) &= \frac{s}{f^2} - a_\eta^2 \frac{s}{f^2} \frac{s}{s - m_\eta^2 + i\Gamma_\eta m_\eta \theta(s)} \\ B(s, t, u) &= 0. \end{aligned} \quad (62)$$

As for the ρ , we have added the imaginary part in the denominator of the above formula to take into account the finite width, Γ_η , of the η . By including only the decay channels to NG bosons, we find:

$$\Gamma_\eta = \frac{a_\eta^2 m_\eta^3}{8\pi f^2}. \quad (63)$$

The expression for $A(s, t, u)$ in eq.(62) coincides with that previously derived in the literature for a scalar singlet of $SO(3)$ in $SO(4)/SO(3)$ theories, see for example Ref. [6]

Figure 3 reports the cross sections of $W_L^+ W_L^- \rightarrow hh$ and $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ for $\xi = 0.5$, $m_\eta = 1.5$ TeV and $a_\eta = 1$, which implies $\Gamma_\eta = 1.1$ TeV. We do not show the cross section of $W_L^\pm Z_L \rightarrow W_L^\pm Z_L$ since it is qualitatively equal to that of $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, as well as $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ and $Z_L Z_L \rightarrow Z_L Z_L$ have the same qualitative behavior of $W_L^+ W_L^- \rightarrow hh$. Notice that $Z_L Z_L \rightarrow Z_L Z_L$ identically vanishes at $O(p^2)$, but can be generated by the η exchange. As expected, the effect of η is that of enhancing those processes, like $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, $W_L^+ W_L^- \rightarrow hh$, $Z_L Z_L \rightarrow Z_L Z_L$, where it is exchanged in s channel, while the others are suppressed compared to the $O(p^2)$ result.

Figure 4 shows how the $W_L^+ W_L^- \rightarrow hh$ cross section is affected by varying a_η . In particular, we find that for $a_\eta < 1$ there is a destructive interference between the η contribution and the $O(p^2)$ term which gives a suppression of the cross section right after the resonance peak. That is similar to the Z -photon interference observed at LEP.

3.3 $\Delta = (\mathbf{3}, \mathbf{3})$

In the case of $SO(5)/SO(4)$, since the broken generators $T^{\hat{a}}$ transform as a $\mathbf{4}$ of $SO(4)$, the product $T^{\hat{a}} T^{\hat{b}}$ can be decomposed into the direct sum $\mathbf{1} + \mathbf{6} + \mathbf{9}$. In particular, the traceless symmetric tensor

$$\left(K^{\hat{a}\hat{b}} \right)_{ij} = \{T^{\hat{a}}, T^{\hat{b}}\}_{ij} - \frac{1}{2} \delta^{\hat{a}\hat{b}} \sum_{\hat{c}=1}^4 (T^{\hat{c}} T^{\hat{c}})_{ij} \quad (64)$$

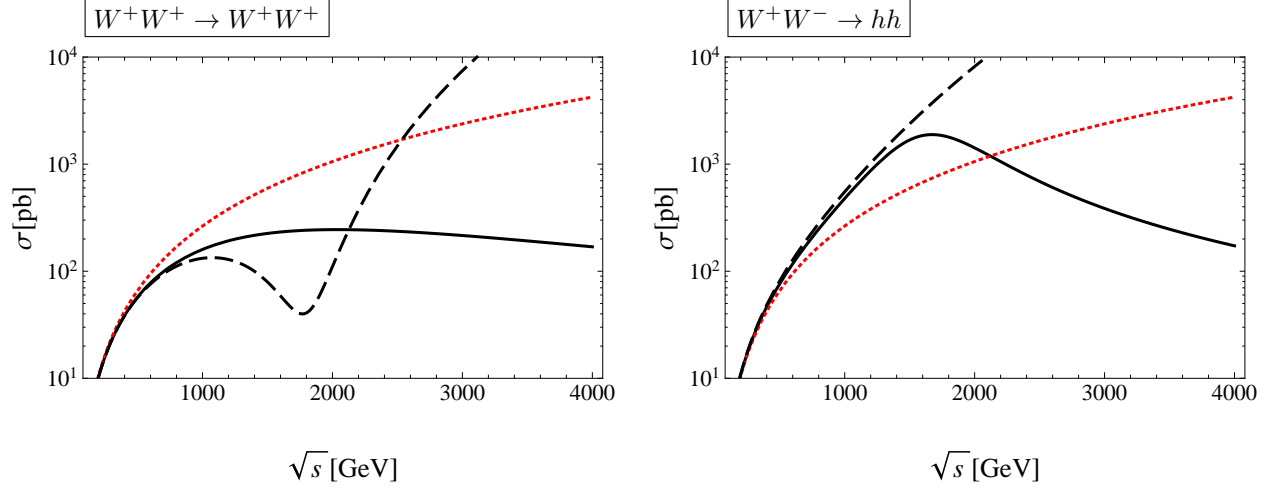


Figure 3: Contribution of η to the $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ (left) and $W_L^+ W_L^- \rightarrow hh$ (right) cross sections for $\xi = 0.5$, $m_\eta = 1.5$ TeV and $a_\eta = 1$, which implies $\Gamma_\eta = 1.1$ TeV. The dotted red and dashed black curves respectively show the $O(p^2)$ and $O(p^4)$ predictions, as obtained from eq.(61). The solid black curve shows the full effect of the η exchange, as computed by means of eq.(62).

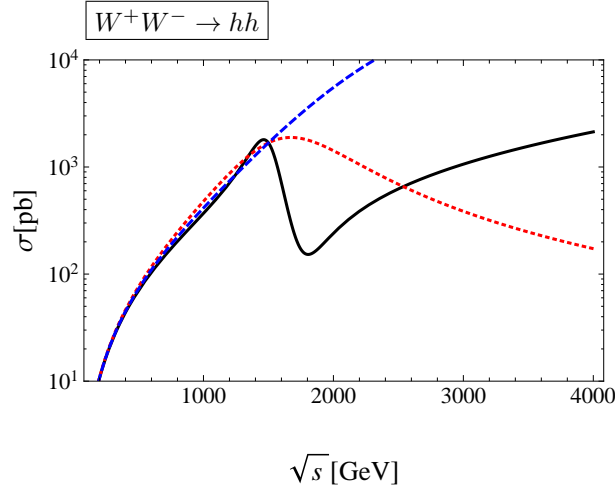


Figure 4: Contribution of η to the $W_L^+ W_L^- \rightarrow hh$ cross section for $a_\eta = 0.5$ (black solid curve), $a_\eta = 1$ (red dotted curve) and $a_\eta = 2$ (blue dashed curve). The other input parameters are fixed as in Fig. 3.

corresponds to the **9** and can be used to describe our scalar Δ . We thus define a 5×5 matrix

$$(\Delta_5(x))_{ij} = \Delta^{\hat{a}\hat{b}}(x) \left(K^{\hat{a}\hat{b}} \right)_{ij}, \quad (65)$$

which contains 9 real scalar fields, denoted as $\Delta^{\hat{a}\hat{b}}(x)$. Under a global transformation $g \in SO(5)$, Δ_5 transforms as

$$\Delta_5 \rightarrow h(\Pi, g) \Delta_5 h^\dagger(\Pi, g). \quad (66)$$

The lagrangian for Δ_5 reads

$$\begin{aligned} \mathcal{L}^{(\Delta)} = & \frac{1}{4} \text{Tr}[(\nabla_\mu \Delta_5)^2] - \frac{1}{4} m_\Delta^2 \text{Tr}[(\Delta_5)^2] + \frac{f^2}{4} \text{Tr} \left[\left(2\sqrt{2} a_\Delta \frac{\Delta_5}{f} + b_\Delta \frac{\Delta_5^2}{f^2} \right) d_\mu d^\mu \right] \\ & + \frac{c_\Delta}{4} \text{Tr}[\Delta_5 d_\mu \Delta_5 d^\mu] , \end{aligned} \quad (67)$$

where a_Δ , b_Δ , c_Δ are free parameters. We have included only the leading operators in ∂/Λ relevant for $\pi\pi$ scattering, and omitted cubic and quartic self-interactions for Δ_5 since they are not relevant for the following. Upon expanding in the NG fields, one obtains the following $\Delta\pi\pi$ interaction term:

$$\mathcal{L}^{(\Delta)} \supset \frac{a_\Delta}{\sqrt{2}f} \left(\sum_a \Delta^{aa} \partial_\mu \pi^a \partial^\mu \pi^a + 2 \sum_{b < a} \Delta^{ab} \partial_\mu \pi^a \partial^\mu \pi^b \right) + \dots \quad (68)$$

Again, requiring that the couplings do not exceed g_* at the cut off scale $g_* f$ implies

$$a_\Delta, b_\Delta, c_\Delta \lesssim O(1) . \quad (69)$$

The weaker request of perturbativity up to the scale $g_* f$ corresponds instead to

$$a_\Delta^2, b_\Delta, c_\Delta \lesssim O\left(\frac{16\pi^2}{g_*^2}\right) . \quad (70)$$

Similarly to the case of the η , also the lagrangian (67) is P_{LR} invariant, with Δ even. This means that the exchange of the Δ does not mediate any P_{LR} -violating process, nor does it generates P_{LR} -odd operators once integrated out at low energy.

As before, at energies $E \ll m_\Delta$ the field Δ_5 can be integrated out by solving the equations of motion at leading chiral order. We find ¹⁴

$$(\Delta_5)_{ij} = \frac{\sqrt{2} a_\Delta f}{m_\Delta^2} \left((d_\mu d^\mu)_{ij} - \frac{1}{4} \sum_{\hat{a}=1}^4 (T^{\hat{a}} T^{\hat{a}})_{ij} \text{Tr}[d_\mu d^\mu] \right) \left(1 + O\left(\frac{E^2}{m_\Delta^2}\right) \right) . \quad (72)$$

Plugging the above solution into eq.(67) and keeping only $O(p^4)$ terms gives a low-energy chiral lagrangian of the form (25) with

$$c_1 = -\frac{1}{4} c_2 = -\frac{a_\Delta^2 f^2}{32 m_\Delta^2} \quad (73)$$

and all the remaining chiral coefficients equal to zero. ¹⁵

¹⁴The following identity is useful to solve the equations of motion:

$$\text{Tr}[K^{\hat{a}\hat{b}} T^{\hat{c}} T^{\hat{d}}] = \frac{1}{2} \text{Tr}[K^{\hat{a}\hat{b}} K^{\hat{c}\hat{d}}] = \frac{1}{4} \left(\delta^{\hat{a}\hat{d}} \delta^{\hat{b}\hat{c}} + \delta^{\hat{a}\hat{c}} \delta^{\hat{b}\hat{d}} - \frac{1}{2} \delta^{\hat{a}\hat{b}} \delta^{\hat{c}\hat{d}} \right) . \quad (71)$$

¹⁵ This result can be compared with the formulas known for the $SO(4)/SO(3)$ case. Indeed, the scalar Δ contains three components, σ , ζ^i , φ^{ij} , respectively with $SO(3)$ isospin 0, 1 and 2. When integrating out a

From eq.(73) and (30) then it follows that, for $E \ll m_\Delta$,

$$\begin{aligned} A(s, t, u) &= \frac{s}{f^2} + \frac{a_\Delta^2}{4m_\Delta^2} (2(t^2 + u^2) - 2s^2) \\ B(s, t, u) &= 0. \end{aligned} \quad (77)$$

Full inclusion of the Δ exchange up to energies $E \ll \Lambda$ gives

$$\begin{aligned} A(s, t, u) &= \frac{s}{f^2} + \frac{a_\Delta^2}{4f^2} \left[\frac{s^2}{s - m_\Delta^2 + i\Gamma_\Delta m_\Delta \theta(s)} - \frac{2t^2}{t - m_\Delta^2 + i\Gamma_\Delta m_\Delta \theta(t)} \right. \\ &\quad \left. - \frac{2u^2}{u - m_\Delta^2 + i\Gamma_\Delta m_\Delta \theta(u)} \right] \\ B(s, t, u) &= 0. \end{aligned} \quad (78)$$

Notice that, contrary to what we have found for the other resonances, Δ cannot unitarize perturbatively the $\pi\pi$ scattering amplitudes for any choice of the input parameters. As before, the imaginary part in the denominators has been added to take into account the finite width, Γ_Δ , of Δ . By including only the decay channels to NG bosons, we find:

$$\Gamma_\Delta = \frac{a_\Delta^2 m_\Delta^3}{32\pi f^2}. \quad (79)$$

Figure 5 reports the cross sections of $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, $W_L^+ W_L^- \rightarrow hh$, $W_L^+ W_L^- \rightarrow W_L^+ W_L^+$ and $W_L^+ Z_L \rightarrow W_L^+ Z_L$ for $\xi = 0.5$, $m_\Delta = 1.5$ TeV and $a_\Delta = 1$ (which gives $\Gamma_\Delta = 277$ GeV). In this case, due to its isospin 0, 1 and 2 components, Δ can be exchanged in s channel in all processes, which are thus all enhanced compared to their $O(p^2)$ value.

heavy resonance in an $SO(4)/SO(3)$ theory, two chiral operators are generated at the $O(p^4)$ level:

$$\bar{c}_1 \text{Tr}[(D_\mu \Sigma^\dagger D^\mu \Sigma)]^2 + \bar{c}_2 \text{Tr}[(D_\mu \Sigma^\dagger D_\nu \Sigma)] \text{Tr}[(D^\mu \Sigma^\dagger D^\nu \Sigma)] , \quad (74)$$

where $\Sigma = \exp(i\sigma^i \chi^i / v)$. These operators are respectively contained in O_1 and O_2 of eq.(25) provided one identifies $\bar{c}_{1,2} = c_{1,2}$ and $f = v$, as it can be checked for example by expanding in the number of NG boson fields. By integrating out the σ and the φ^{ij} in an $SO(4)/SO(3)$ theory we find

$$c_1|_\sigma = + \frac{4}{3} \pi \frac{v^4}{m_\sigma^5} \Gamma_\sigma \quad c_2|_\sigma = 0 \quad (75)$$

$$c_1|_\varphi = - \frac{4}{3} \pi \frac{v^4}{m_\varphi^5} \Gamma_\varphi \quad c_2|_\varphi = - 3c_1|_\varphi , \quad (76)$$

while the ζ^i does not contribute. In the $SO(5)/SO(4)$ case, the σ component inside Δ also couples to the Higgs (according to eq.(67)), and its decay width gets enhanced by a factor 4 compared to the $SO(4)/SO(3)$ case. The coefficients in (73) are thus obtained as the sum of the σ and φ contributions in eqs.(75),(76) after replacing $\Gamma_\sigma \rightarrow (1/4)\Gamma_\sigma$ in $c_1|_\sigma$ and setting $m_\sigma = m_\varphi = m_\Delta$, $\Gamma_\sigma = \Gamma_\varphi = \Gamma_\Delta$. Equation (75) agrees with the $SO(4)/SO(3)$ result for σ reported in Ref. [5], but eq.(76) disagrees with the formulas given in the same paper for the isospin tensor φ .

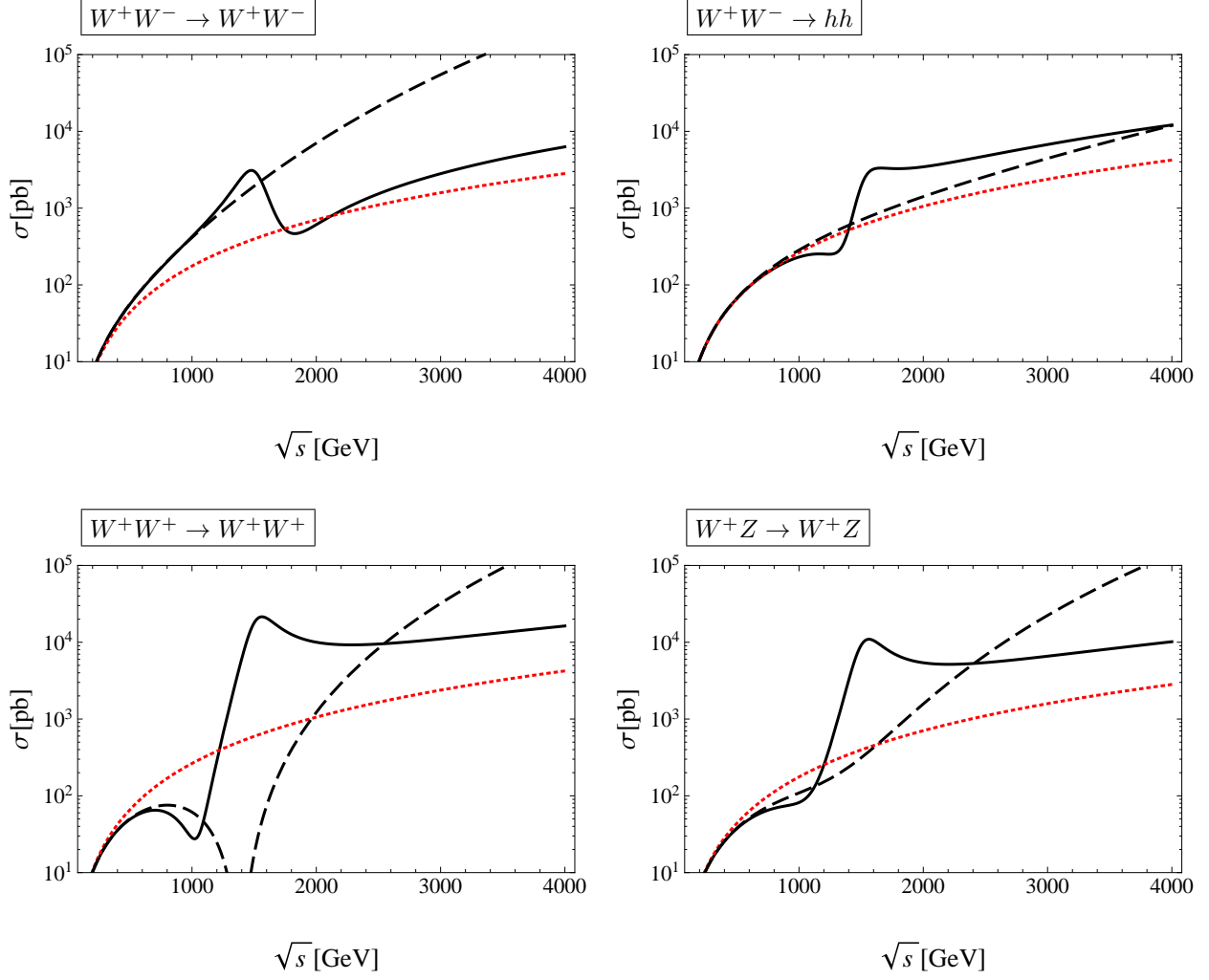


Figure 5: Contribution of Δ to the $W_L^+W_L^- \rightarrow W_L^+W_L^-$ (upper left), $W_L^+W_L^- \rightarrow hh$ (upper right), $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ (lower left) and $W_L^+Z_L \rightarrow W_L^+Z_L$ (lower right) cross sections for $\xi = 0.5$, $m_\Delta = 1.5$ TeV and $a_\Delta = 1$, which implies $\Gamma_\Delta = 277$ GeV. The dotted red and dashed black curves respectively show the $O(p^2)$ and $O(p^4)$ predictions, as obtained from eq.(77). The solid black curve shows the full effect of the Δ exchange, as computed by means of eq.(78).

4 WW scattering at the LHC

In this section we study the effect of the resonances on WW scattering processes at the LHC. For simplicity we will make use of the Effective W Approximation (EWA) [29], according to which a Vector Boson Fusion (VBF) process can be effectively described as the slow emission of two vector bosons from the partons inside the protons, which subsequently undergo a (fast) hard scattering. This picture neglects the small degree of off-shellness of the emitted vector bosons (of the order of their transverse momentum), and thus allows one to compute the differential cross section for the VBF process $pp \rightarrow X + jj$ as the convolution of a partonic

cross section $\hat{\sigma}(V_i V_j \rightarrow X)$ times a luminosity factor $\rho_{V_i V_j}(m_X^2/s, Q^2)$:

$$\begin{aligned} \frac{d\sigma}{dm_X^2} &= \frac{1}{m_X^2} \hat{\sigma}(V_i V_j \rightarrow X) \rho_{V_i V_j}(m_X^2/s, Q^2) \\ \rho_{V_i V_j}(\tau, Q^2) &= \tau \int_0^1 dx_1 \int_0^1 dx_2 f_{q_A}(x_1, Q^2) f_{q_B}(x_2, Q^2) \times \\ &\quad \times \int_0^1 dz_1 \int_0^1 dz_2 P_{q_A}^i(z_1) P_{q_B}^j(z_2) \delta(x_1 x_2 z_1 z_2 - \tau). \end{aligned} \quad (80)$$

An implicit sum over all partons q_A, q_B and over transverse and longitudinal vector boson polarizations $i, j = T, L$ is understood. In our case the final states of interest are $X = V_L V_L, hh$, with $V_L = W_L, Z_L$. The luminosity factor $\rho_{V_i V_j}(\tau, Q^2)$ is defined as the probability for a pair of vector bosons $V_i V_j$ with invariant mass $m_X^2 = \tau s$ to be emitted from the two colliding protons [30], where $\sqrt{s} = 14 \text{ TeV}$ is the LHC center of mass energy and Q is the factorization scale. As shown in eq.(80), it can be computed by integrating over the quark PDF's $f_q(x, Q^2)$, and the vector boson splitting functions $P_q^i(z)$. These latter are given by [29]

$$\begin{aligned} P^T(z) &= \frac{g_A^2 + g_V^2}{4\pi^2} \frac{1 + (1-z)^2}{2z} \log \left[\frac{\bar{p}_T^2}{(1-z)m_W^2} \right] \\ P^L(z) &= \frac{g_A^2 + g_V^2}{4\pi^2} \frac{1-z}{z}, \end{aligned} \quad (81)$$

where, z indicates the fraction of energy carried by the vector boson and \bar{p}_T is the largest value allowed for its transverse momentum. They depend upon the parton flavor and the vector boson species through the vectorial and axial couplings g_V, g_A :

$$\begin{aligned} (g_V)_{qW} &= -(g_A)_{qW} = \frac{g}{2\sqrt{2}} & (g_V)_{uZ} &= \frac{g}{\cos \theta_W} \left(\frac{1}{4} - \frac{2}{3} \sin^2 \theta_W \right) \\ (g_A)_{uZ} &= -(g_A)_{dZ} = \frac{g}{4 \cos \theta_W} & (g_V)_{dZ} &= \frac{g}{\cos \theta_W} \left(-\frac{1}{4} + \frac{1}{3} \sin^2 \theta_W \right). \end{aligned} \quad (82)$$

By making use of eq.(80) one can easily compute the total cross section for each final state of interest by integrating over all values of m_X . As a further simplification, we neglected the contribution from the transverse vector bosons and we used the expressions of the longitudinal cross sections derived in the previous section at leading chiral order by means of the Equivalence Theorem. We expect this latter approximation to work only at sufficiently large m_X , a limit in which the contribution of the longitudinal polarizations dominates and the vector bosons can be effectively treated as massless NG bosons. Restricting to events with large m_X is also required in order to reduce the model-dependent effects in $V_L V_L \rightarrow hh$ due to the Higgs cubic self-coupling, as shown by the analysis of Ref. [30].¹⁶ We thus imposed the following cut:

$$m_X \geq m_{\text{cut}} = 800 \text{ GeV}. \quad (83)$$

¹⁶Non-derivative couplings involving h and η (or Δ), which are induced by explicit $SO(5)$ breaking from SM couplings, are also a source of model dependency. At around the η (or Δ) mass their relative importance is only $O(g_{SM}^2/g_\rho^2)$, which can be safely neglected.

A smaller value of the cut is still appropriate for the $V_L V_L$ final states, but it does not change substantially our conclusions. We will discuss in detail the dependence of our results on m_{cut} in the following.

In order to monitor the effect of a light resonance $\Phi = \eta, \rho, \Delta$, we construct the ratio

$$R(\Phi, \xi, m_{\text{cut}}) = \frac{\sigma(\Phi, \xi, m_{\text{cut}})}{\sigma(\text{LET}, \xi, m_{\text{cut}})} \quad (84)$$

for each process of interest, where $\sigma(\Phi, \xi, m_{\text{cut}})$ fully includes the contribution of the resonance while $\sigma(\text{LET}, \xi, m_{\text{cut}})$ is obtained by computing the partonic cross section using the $SO(5)/SO(4)$ chiral lagrangian at $O(p^2)$. The ratio is largely insensitive to the effect of the acceptance cuts on the forward jets which will be applied in a realistic analysis and that have not been included in the calculation of the individual cross sections by means of eq.(80). This directly follows from the fact that the slow emission of the vector bosons factorizes from the subsequent hard scattering, hence the kinematic distributions of the forward jets are universal. As discussed in more detail in the following, we have estimated the efficiency for a standard set of acceptance cuts,

$$p_T(j) > 30 \text{ GeV}, \quad |\eta(j)| \leq 5, \quad \Delta R(jj) \geq 0.7, \quad (85)$$

by utilizing the event generator MadGraph [31]. We found $\epsilon_{\text{acc}} \sim 0.6 - 0.45$ in the range $m_{\text{cut}} \sim 0 - 3 \text{ TeV}$ (see Fig. 9 below).

As already stated before, we shall focus on the more restrictive hypothesis of Partial UV Completion where the couplings do not exceed g_* at the cut-off scale $g_* f$. According to PUVC the contribution of each resonance, $\Phi = \{\rho, \eta, \Delta\}$, is controlled at leading order by two parameters: its mass, m_Φ , and its coupling to two NG bosons, a_Φ . For each resonance Φ we will thus show four contour plots in the plane (a_Φ, m_Φ) reporting the ratio $R(\Phi, \xi, m_{\text{cut}})$ for the processes $pp \rightarrow Xjj$ with $X = W_L^+ W_L^-, hh, W_L^+ W_L^+, W_L^+ Z_L$. We impose the cut of eq.(83) and fix $\xi = (v/f)^2 = 0.5$.¹⁷ For these values, in the absence of new resonances (or $m_\Phi \rightarrow \infty$), we obtain:

	process			
	$W_L^+ W_L^-$	hh	$W_L^+ W_L^+$	$W_L^+ Z_L$
$\sigma(\text{LET}, \xi, m_{\text{cut}}) [\text{fb}]$	2.46	2.37	1.87	1.47

which can be used as reference values for a quick estimate of the absolute cross sections for given values of the ratio R . In each contour plot we superimpose the isocurves of constant Γ_Φ/m_Φ , whose value can be used to monitor the strength of the coupling of the resonance to the NG bosons: the region $\Gamma_\Phi/m_\Phi \gtrsim \pi$ is where the coupling $\Phi\pi\pi$ grows strong and our perturbative calculation cannot be trusted.

Figure 6 shows the results for a spin-1 resonance ρ^L . The solid black and dashed blue lines denote respectively the isocurves of constant $R(\rho^L, \xi, m_{\text{cut}})$ and constant $\Gamma_{\rho^L}/m_{\rho^L}$ in the plane (a_{ρ^L}, m_{ρ^L}) , where a_{ρ^L} is defined in eq.(35). The region below the thick green

¹⁷As for the quark PDF's, we used the CTEQ611 set and fixed the factorization scale to $Q = m_W$.

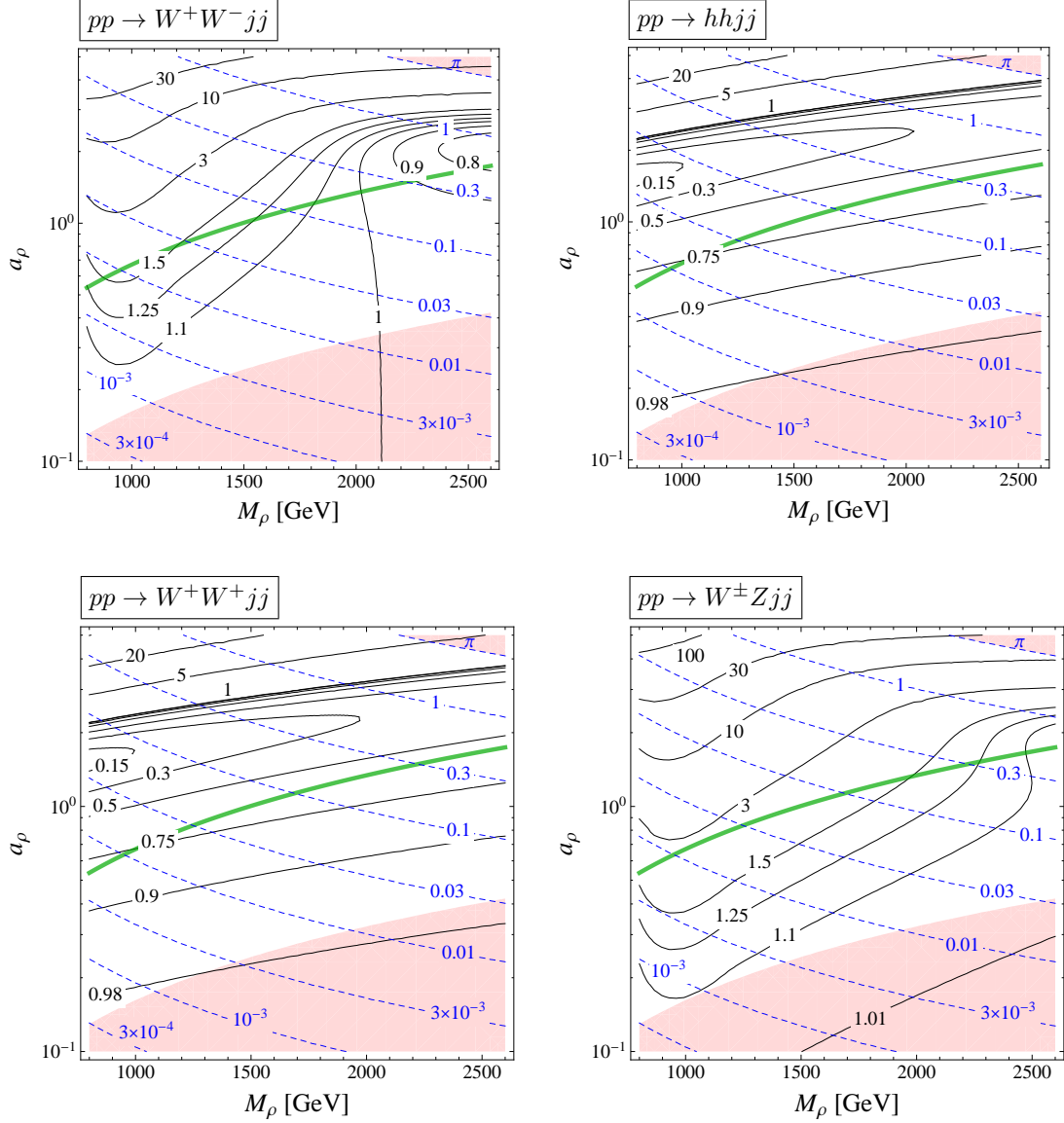


Figure 6: Contours of constant $R(\rho^L, \xi, m_{\text{cut}})$ (continuous black lines) and constant $\Gamma_{\rho_L}/m_{\rho_L}$ (dashed blue lines) in the plane (a_{ρ_L}, m_{ρ_L}) for $\xi = 0.5$ and $m_{\text{cut}} = 800$ GeV. The pink (darker) areas in the upper right corner and in the lower part of the plane respectively correspond to the regions where $\Gamma_{\rho_L}/m_{\rho_L} > \pi$ and $g_{\rho_L} > 4\pi$. In these regions our perturbative calculation cannot be trusted, see text. The region below the thick green line is that where $\Delta\hat{S} < 2.9 \times 10^{-3}$ for $\alpha_2 = 0$ (see text).

line is that where $\Delta\hat{S}$, as computed by means of eq.(39) with $\alpha_2 = 0$, satisfies the bound $\Delta\hat{S} < 2.9 \times 10^{-3}$.¹⁸ In most of the region shown the ρ^L has a width small compared to its mass. In the pink (darker) region in the upper right corner of the plane, however, one has $\Gamma_{\rho_L}/m_{\rho_L} > \pi$, which indicates that the $\pi\pi\rho$ coupling is non-perturbative and consequently

¹⁸This is the bound implied at 99% CL by the LEP electroweak data on $\Delta\hat{S}$ in presence of an arbitrary $\Delta\hat{T}$ correction, as obtained using the EW fit of Ref. [32] with $m_h = 120$ GeV.

our calculation cannot be trusted. The other pink (darker) area in the lower part of the plane corresponds instead to the region where $g_{\rho_L} > 4\pi$ and perturbation theory also breaks down. These two bounds represent the zeroth order request of the viability of our effective lagrangian, and are weaker than any of the two criteria discussed at the beginning of section 3. On the other hand, PUVC singles out the region around $a_\rho \sim 1$. Points further away from $a_\rho \sim 1$ can still be allowed if one adopts the first criterion discussed at the beginning of section 3. However, as discussed in section 3.1, one should bear in mind that in that case neglecting the contribution of the operator Q_1 to the scattering amplitudes, as done in our calculation, is justified only for $g_*^2 < 4\pi g_{\rho_L}$.

In the case of $pp \rightarrow W_L^+ W_L^- jj$ and $pp \rightarrow W_L^+ Z_L jj$, the ratio R gets larger if one fixes m_{ρ_L} and increases a_{ρ_L} . This is expected, considering that in these processes ρ^L can be exchanged in the s -channel and thus tends to increase the cross section, as previously discussed. In the case of $pp \rightarrow W_L^+ W_L^+ jj$ and $pp \rightarrow hhjj$, on the other hand, in the region of small a_{ρ_L} the ratio R decreases if a_{ρ_L} is increased. As also discussed in the previous section, this is the effect of the exchange of the ρ^L in the t - and u -channels, which tends to unitarize the scattering and thus decreases the cross section. Such behavior however changes if one moves to values $a_{\rho_L} \gg 2/\sqrt{3}$: in this case the contribution of ρ^L dominates the scattering amplitude (it ‘overshoots’ the $O(p^2)$ term and no longer unitarizes), thus quickly driving the cross section to larger values. Summarizing, for reference values $m_{\rho_L} = 1.5 \text{ TeV}$, $a_{\rho_L} = 1$, we predict a suppression $R = 0.7$ in the $W_L^+ W_L^+$, hh channels, and an enhancement $R = 1.3$ and $R = 2.0$ respectively in the channels $W_L^+ W_L^-$, $W_L^+ Z_L$.

The ratio R depends upon ξ uniquely through the resonance’s decay width, since both the scattering amplitude at $O(p^2)$ and the resonance contribution are proportional to ξ for fixed a_ρ and m_{ρ_L} , see eq.(53). This property also holds in the case of the other resonances. As a consequence, the dependence of R upon ξ is absent in those processes where the resonance is exchanged only in t and u channel, whereas it can be significant when a resonance exchanged in s -channel is light. In the latter case, for sufficiently narrow width the dependence on the parton distributions drops and one basically has $R \propto 1/\xi$. For sizeable ξ this scaling is partly compensated by the “spreading” of the resonance to $s < m_\rho^2$ where the PDF’s grow steeply. This is clearly illustrated by the following tables, which report the value of R in the channels hh (left) and $W_L^+ Z_L$ (right) for $a_{\rho_L} = 2/\sqrt{3}$:

hh		$m_{\rho_L} [\text{GeV}]$			$W_L^+ Z_L$		$m_{\rho_L} [\text{GeV}]$		
	R	1500	2000	2500		R	1500	2000	2500
	0.1	0.59	0.71	0.78		0.1	8.8	3.7	2.0
ξ	0.5	0.59	0.71	0.78	ξ	0.5	2.3	1.4	1.1
	0.8	0.59	0.71	0.78		0.8	1.6	1.1	1.0

Figure 7 shows the results for the η . In this case the contribution of the resonance tends to enhance the channels hh , $W_L^+ W_L^-$ and to suppress $W_L^+ W_L^+$, $W_L^+ Z_L$. Compared to the ρ , the η has a larger decay width, which implies a more extended region where $\Gamma_\eta/m_\eta > \pi$ and the coupling $\pi\pi\eta$ gets non-perturbative (it corresponds to the pink (darker) area in

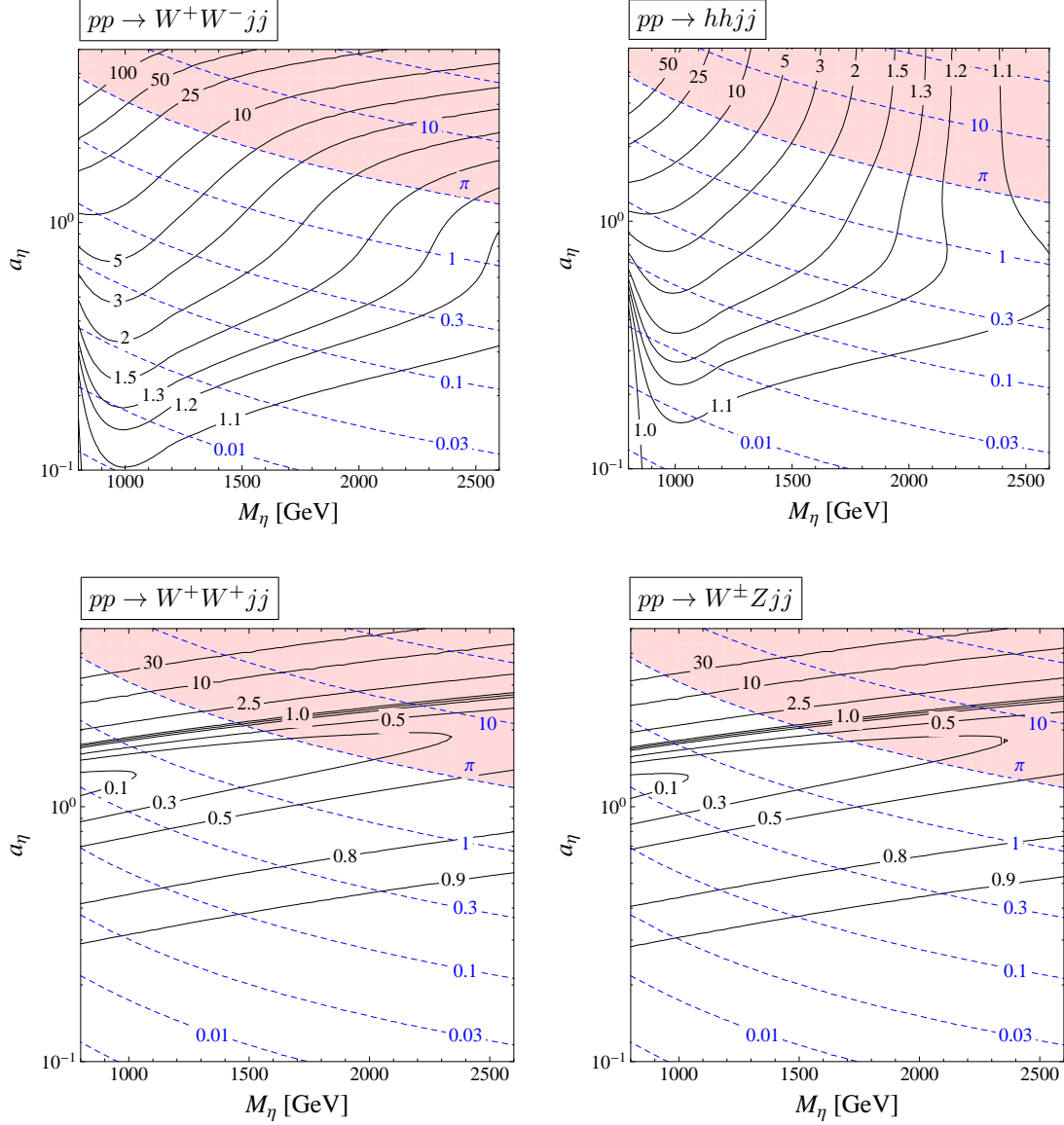


Figure 7: Contours of constant $R(\eta, \xi, m_{\text{cut}})$ (continuous black lines) and constant Γ_η/m_η (dashed blue lines) in the plane (a_η, m_η) for $\xi = 0.5$, $m_{\text{cut}} = 800$ GeV. In the pink (darker) region $\Gamma_\eta/m_\eta > \pi$, and our perturbative calculation cannot be trusted, see text.

Fig. 7). For reference values $m_\eta = 1.5$ TeV, $a_\eta = 1$, we predict a suppression $R = 0.5$ and $R = 0.4$ respectively in the $W_L^+W_L^+$ and $W_L^+Z_L$ channels, and an enhancement $R = 2.9$, $R = 1.9$ respectively in $W_L^+W_L^-$ and hh . As expected from the above discussion, varying ξ does not change the ratio R in the case of the final states $W_L^+W_L^+$, $W_L^+Z_L$, while it has an important effect for hh , $W_L^+W_L^-$, where the resonance is exchanged in s -channel. This is clearly illustrated by the following tables, which report the value of R in the channels hh (left) and $W_L^+W_L^+$ (right) for $a_\eta = 1$:

hh	m_η [GeV]				$W_L^+ W_L^+$	m_η [GeV]			
	R	1500	2000	2500		R	1500	2000	2500
ξ	0.1	8.4	3.6	2.1	ξ	0.1	0.45	0.59	0.69
	0.5	1.9	1.3	1.1		0.5	0.45	0.59	0.69
	0.8	1.4	1.1	1.0		0.8	0.45	0.59	0.69

Finally, the results for the resonance Δ are shown in Fig. 8. In this case, given the quantum numbers of the Δ , the exchange of the resonance tends to enhance all the channels. As for the η , our calculation cannot be trusted in the pink (darker) area of the (m_Δ, a_Δ) region shown in Fig. 8, where the $\pi\pi\Delta$ coupling is non-perturbative ($\Gamma_\Delta/m_\Delta > \pi$). For reference values $m_\Delta = 1.5$ TeV, $a_\Delta = 1$, we predict an enhancement in all channels as follows: $R = 2.3$ in $W_L^+ W_L^-$; $R = 1.7$ in hh ; $R = 6.6$ in $W_L^+ W_L^+$; $R = 5.3$ in $W_L^+ Z_L$. An increase of ξ implies a suppression of the ratio R in all the channels. This is illustrated by the following tables, which report the value of R in the channels hh (left) and $W_L^+ W_L^+$ (right) for $a_\Delta = 1$:

hh	m_Δ [GeV]				$W_L^+ W_L^+$	m_Δ [GeV]			
	R	1500	2000	2500		R	1500	2000	2500
ξ	0.1	3.2	1.7	1.2	ξ	0.1	33	11	4.3
	0.5	1.7	1.3	1.1		0.5	6.6	2.4	1.2
	0.8	1.6	1.2	1.1		0.8	4.3	1.7	1.0

4.1 Check of the analytic approximation

At this point we would like to discuss the precision of our approximate analytic calculation of the cross sections. We will do so by comparing with a full calculation performed by using a Montecarlo simulation. We consider the models MCHM4 and MCHM5 defined in Refs. [2] and [16], and their linearized versions, which we will denote respectively as LMCHM4 and LMCHM5 in the following. The LMCHM4, in particular, has been already considered in Ref. [19]. In both linearized models a scalar resonance η is added to the original $SO(5)/SO(4)$ chiral lagrangian to form a linear representation of $SO(5)$ together with the four NG bosons. The lagrangian which describes the derivative interactions between η and the NG bosons is that of eq.(55) with $a_\eta = b_\eta = 1$. The two linearized models thus differ only by (non-derivative) potential terms, as well the original MCHM4 and MCHM5 differ only by the Higgs trilinear coupling. The reader can find all the details and the relevant formulas in Appendix G.

We first checked that the efficiency of the acceptance cuts defined in eq.(85) is universal, *i.e.* it does not depend on the model, as prescribed by the EWA. For simplicity, we focused on the process $pp \rightarrow hhjj$ and set $m_h = 180$ GeV,¹⁹ so that our results partly extend the

¹⁹For this value of the Higgs mass, both in the MCHM4 and MCHM5 as well as in their linearized versions,

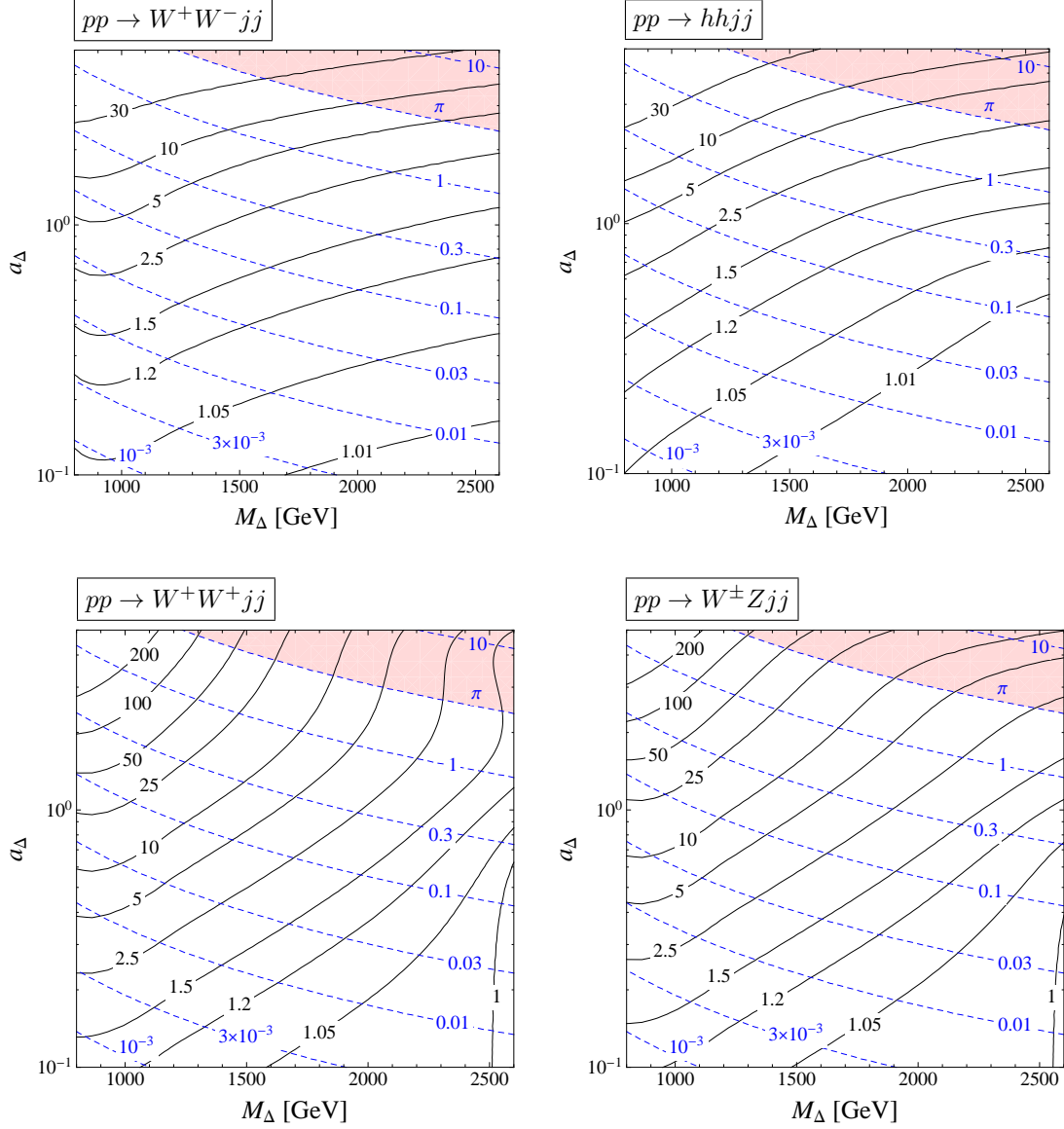


Figure 8: Contours of constant $R(\Delta, \xi, m_{\text{cut}})$ (continuous black lines) and constant $\Gamma_{\Delta}/m_{\Delta}$ (dashed blue lines) in the plane (a_{Δ}, m_{Δ}) for $\xi = 0.5$ and $m_{\text{cut}} = 800$ GeV. In the pink (darker) region $\Gamma_{\Delta}/m_{\Delta} > \pi$, and our perturbative calculation cannot be trusted, see text.

analysis of Ref. [30]. We used the Montecarlo program MadGraph [31] to derive the cross sections in the models MCHM4 and MCHM5 and in their linearized versions.²⁰ Figure 9 reports the efficiency of the acceptance cuts, ϵ_{acc} , as a function of the value of m_{cut} in the MCHM4 and in the LMCHM4 for $m_{\eta} = 1.5$ TeV and $m_{\eta} = 2.0$ TeV. In all the models ξ has

the region at small ξ is excluded by the LHC results recently presented at the 2011 Lepton Photon Conference. As explained below in the text, in this section we choose $\xi = 0.5$, which is presently not excluded by any Higgs search.

²⁰Like in the previous section, we used the set of PDFs CTEQ611 and fixed the factorization scale to $Q = m_W$.

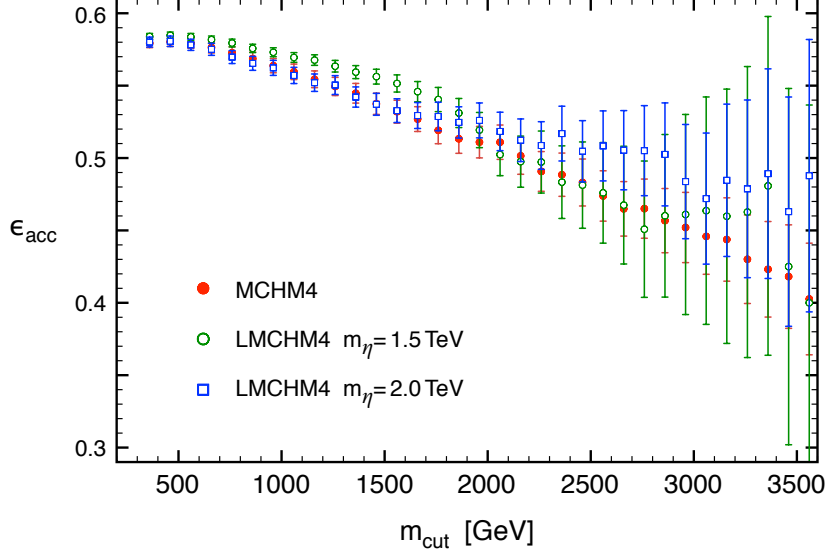


Figure 9: Efficiency of the acceptance cuts of eq.(85) as a function of m_{cut} for the process $pp \rightarrow hhjj$ (with $m(hh) \geq m_{\text{cut}}$). Filled red circles, empty green circles and empty blue squares respectively correspond to the MCHM4, the LMCHM4 with $m_\eta = 1.5 \text{ TeV}$, the LMCHM4 with $m_\eta = 2.0 \text{ TeV}$. In all the models $\xi = 0.5$. The vertical bars report the statistical theoretical error in each point.

been set to 0.5. The efficiency is the same in the three models within the statistical error. We find that the same conclusion holds in the MCHM5 and its linearized version, and when ξ is varied. The decrease of ϵ_{acc} with m_{cut} is a simple kinematic effect: for fixed center of mass energy \hat{s} of the quarks initiating the scattering, the outgoing jets tend to be softer for larger values of m_{cut} , hence the efficiency of the acceptance cuts falls off.

The second issue to address is how well our analytic calculation of the cross section, in which we neglected the effect of non-derivative couplings and relied on the Equivalence Theorem, reproduces the full result. As already discussed, we expect the agreement to be good for large values of m_{cut} , whereas the contribution from non-derivative couplings and the transverse polarizations of the vector bosons should become important approaching the threshold. This expectation is indeed confirmed by comparing the Montecarlo and the analytic results for the cross section: the plots of Figures 10 and 11 show that while in the MCHM4 and its linearized versions the agreement turns out to be at the level of 5% or better, except for points right at threshold, in the case of the (L)MCHM5 the analytic curve differs from the Montecarlo prediction by up to 30 – 40% for $m_{\text{cut}} \lesssim 1 \text{ TeV}$. In other words, we find that the model dependency due to non-derivative couplings starts to be important at energies $m_{hh} \sim 1 \text{ TeV}$, in agreement with what found by Ref. [30]. Interestingly, part of this model dependency cancels out in taking the ratio of cross sections, so that R is better reproduced by the analytic calculation. Figures 12 and 13 show that the agreement is better than 10%, even for values of the cut near threshold, both in the LMCHM4 and the LMCHM5. We thus conclude that R is a quite robust quantity to monitor the importance of the resonance contribution.

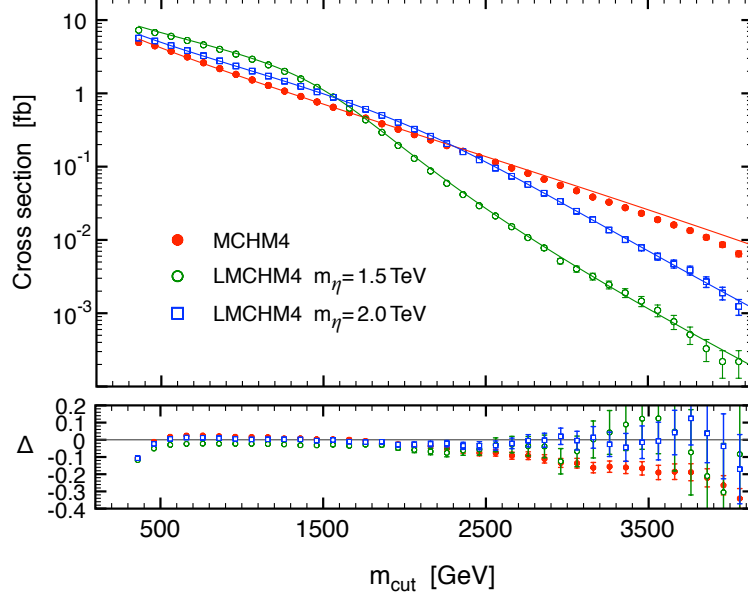


Figure 10: Upper panel: Cross section of $pp \rightarrow hhjj$ as a function of the cut on m_{hh} . Filled red circles, empty green circles and empty blue squares respectively correspond to the MCHM4, the LMCHM4 with $m_\eta = 1.5$ TeV, the LMCHM4 with $m_\eta = 2.0$ TeV. The continuous curves denote the corresponding analytic results. Lower panel: relative difference between the Montecarlo and the analytic predictions, $\Delta = (\sigma(MC) - \sigma(analytic))/\sigma(analytic)$, as a function of the cut on m_{hh} . In all the models $\xi = 0.5$. The vertical bars report the statistical theoretical error in each point.

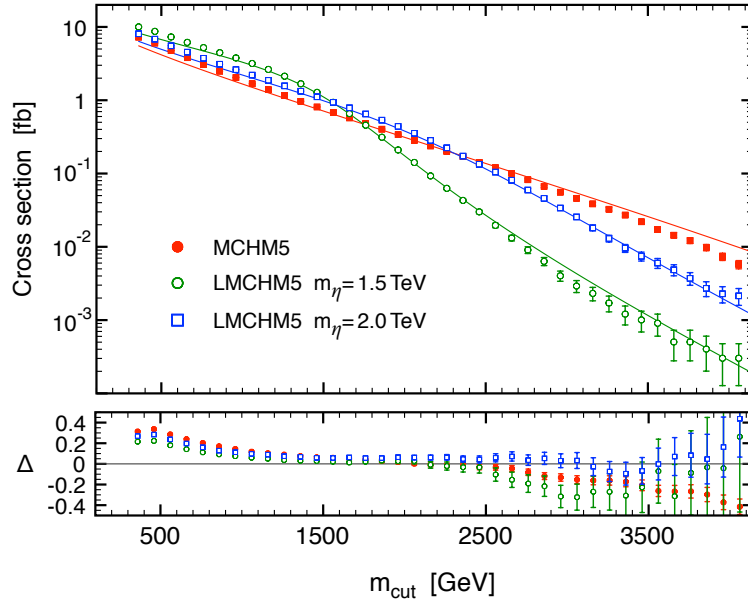


Figure 11: As in Fig. 10 but for the MCHM5 and its linearized version.

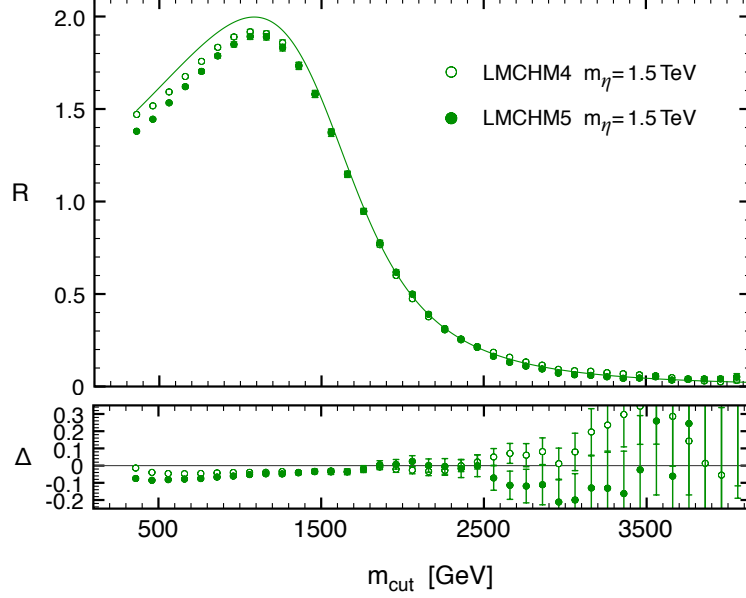


Figure 12: Upper panel: Ratio $R(\eta, \xi = 0.5, m_{\text{cut}})$ for the process $pp \rightarrow hhjj$ as a function of the cut on m_{hh} . Empty (filled) green circles correspond to the LMCHM4 (LMCHM5) with $m_\eta = 1.5 \text{ TeV}$. The continuous curve denotes the analytic result. Lower panel: relative difference between the Montecarlo and the analytic prediction, $\Delta = (R(MC) - R(\text{analytic}))/R(\text{analytic})$, as a function of the cut on m_{hh} . The vertical bars report the statistical theoretical error in each point.

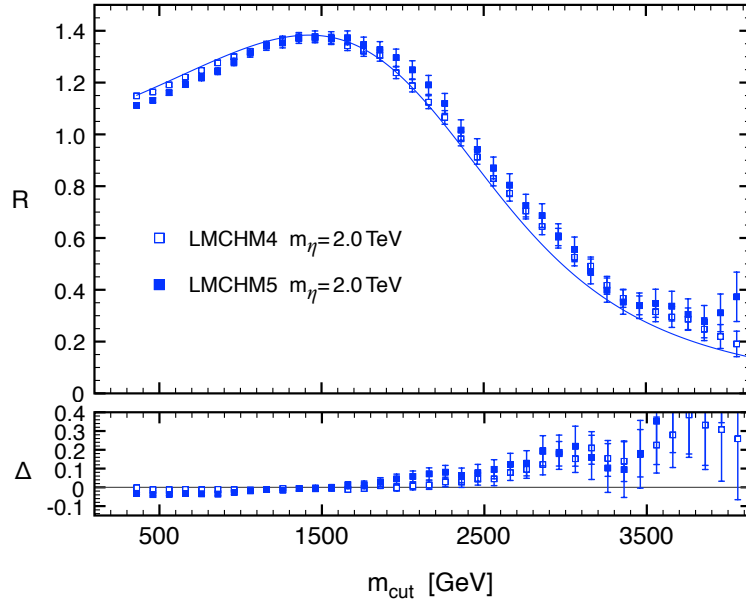


Figure 13: Upper panel: Ratio $R(\eta, \xi = 0.5, m_{\text{cut}})$ for the process $pp \rightarrow hhjj$ as a function of the cut on m_{hh} . Empty (filled) blue squares correspond to the LMCHM4 (LMCHM5) with $m_\eta = 2.0 \text{ TeV}$. The continuous curve denotes the analytic result. Lower panel: relative difference between the Montecarlo and the analytic prediction, $\Delta = (R(MC) - R(\text{analytic}))/R(\text{analytic})$, as a function of the cut on m_{hh} . The vertical bars report the statistical theoretical error in each point.

5 Summary

In this paper we have extended the study of the phenomenology of a general $SO(5)/SO(4)$ composite Higgs model beyond the leading $O(p^2)$ chiral lagrangian. We have done that along two different, but related, approaches.

Our first approach consisted in the study of the general lagrangian at $O(p^4)$. In doing that we have classified the relevant discrete symmetries, accidental or approximate, and discussed their possible phenomenological role. First of all we have pointed out R , the grading Z_2 symmetry of $SO(5)/SO(4)$ under which the broken generators are odd while the unbroken ones are even. Since $R \subset SO(4)$, it is exact to all orders in the chiral lagrangian in the limit in which the SM couplings are neglected. One consequence is that no processes involving an odd number of Goldstones can be significantly enhanced by the strong interaction, even after taking into account a non-vanishing Higgs expectation value. In particular, the process $W_L^+ W_L^- \rightarrow hhh$ vanishes at leading order in the SM gauge couplings [33]. Another relevant symmetry is P_{LR} , the exchange of the L and R components of $SU(2)_L \times SU(2)_R \sim SO(4)$. We have shown that P_{LR} is an accidental symmetry up to $O(p^2)$ and is broken by several $O(p^4)$ operators. Moreover, the leading effect of P_{LR} breaking in $\pi\pi$ scattering can only arise at $O(p^6)$ in the chiral lagrangian. Such a term could be induced at tree level by the exchange of a vector resonance either in the $(3, 1)$ or in the $(1, 3)$ of $SO(4)$, of course provided the spectrum breaks P_{LR} . Such a possibility is perfectly sensible although often disregarded in explicit 5D constructions. However, because of the $O(p^6)$ dependence, the potentially interesting P_{LR} -breaking process $W_L^+ W_L^- \rightarrow Z_L h$ is very suppressed at low energy, and can only become relevant at the LHC if a P_{LR} -breaking vector resonance happens to be rather light. Finally, another relevant symmetry is Higgs parity $P_h \equiv R \times P_{LR}$. The validity of P_h at the chiral lagrangian level coincides with the validity of P_{LR} . However, for the specific case where $v = f$, P_h survives in the bosonic sector even after gauging $SU(2)_L \times U(1)_Y$. In the limit where the SM couplings are turned off, P_h is valid under the same conditions as P_{LR} , since $R \subset SO(4)$ is exact.

In our second approach we modeled the lagrangian of possible low-lying resonances and studied their effects. Our picture is that there may be some resonances that are lighter than the “bulk” of the spectrum and that their effect may be reasonably well described by a phenomenological lagrangian. In order to formalize that picture we have worked under the assumption that the bulk of the spectrum is broadly characterized by a coupling g_* , and a mass scale $\Lambda \sim g_* f$, while the low-lying resonance Φ has mass m_Φ parametrically smaller than Λ . We have introduced a principle, that we call *partial UV completion* (PUVC), to parametrize the couplings of such Φ . The simplest way to phrase PUVC is that it corresponds to the request that all couplings be of order g_* at the scale Λ . The main practical consequence of PUVC is that the contribution of Φ in $\pi\pi$ scattering is well described by just m_Φ and a coupling g_Φ such that $a_\Phi \equiv m_\Phi/g_\Phi f \sim 1$.²¹ One way to see that PUVC is sensible is by noticing that it corresponds to the range of coupling and mass such that the interaction among NG bosons is not made stronger by the presence of the light resonance. Moreover, within the range of parameters dictated by PUVC, one can accommodate models that reduce

²¹One possible explicit realization of PUVC in the parametric limit $g_\Phi \ll g_*$ can be obtained in 5D constructions by introducing large kinetic terms on the IR brane. In that case g_* corresponds to the KK coupling, while g_Φ is controlled by the boundary kinetic term.

over a certain range the strength of the coupling among NG bosons. In this sense PUVC generalizes to a broader set of models the restrictive parameter choice of theories where the lighter resonance is required to exactly unitarize $\pi\pi$ scattering. Our description is strictly speaking perturbative and consistent as long as $g_\Phi \ll g_*$. In particular, for the case $g_* \sim 4\pi$ we need $g_\Phi \ll 4\pi$. Nonetheless the hope is that our parametrization could be qualitatively correct in realistic and genuinely strongly coupled cases. That is in analogy with other situations, like large N_c in QCD, where the insights of an approximation seemingly extend beyond its expected range of validity.

We have applied our parametrization to study resonances of spin 0 and 1 that can affect $\pi\pi$ scattering. For spin 0, Bose symmetry dictates that we can either have a $\eta \equiv (1, 1)$ of $SO(4)$ or a $\Delta \equiv (3, 3)$. In the spin-1 channel we can instead have either $\rho_L \equiv (3, 1)$ or $\rho_R \equiv (1, 3)$. Two broad properties should be remarked. The first, perhaps not obvious, is that according to PUVC vector resonances are narrower, relative to their mass, than the scalar ones. That is even more true when comparing ρ_L or ρ_R to the total singlet η . This fact was already remarked before [34] and is compatible with the situation in QCD, where the σ is relatively much broader than the ρ . The narrowness of the vectors makes them suitable for discovery in Drell-Yan production. Indeed a_ρ , m_ρ and possibly α_2 offer a simple, and theoretically robust, parametrization of the Drell-Yan production and decay of composite vectors. That seems ideal to express the results of the searches at the LHC. The second and more obvious property is that each resonance enhances those processes where it is exchanged in the s -channel while it depletes those in which it is exchanged only in the t - and u -channels. Thus $\rho_{L,R}$ exchange enhances the cross section for W^+W^- and WZ final states and depletes the one in hh and W^+W^+ . Exchange of the total singlet η enhances W^+W^- and hh and reduces the others. Finally, the tensor scalar Δ appears in the s -channel for all processes, and thus enhances them all.

Aside these qualitative features it turns out that the effect of the scalars is quantitatively more significant at comparable value of the mass and of the corresponding coefficient a_Φ . Indeed, as already noticed for the case of technicolor, the exchange of a vector resonance does not significantly change the $pp \rightarrow VVjj$ cross section: less than a factor of 2 over a broad range of parameters, and this is even more true in the region which is favored by the bound on the S parameter. More precisely, focussing on the case where $\xi \gtrsim 0.3$ for which there is some chance to study WW scattering at the LHC, one can synthesize the results as follows (see Figs. 6, 7, 8). For $m_\Phi \gtrsim 2$ TeV the cross section is modified by less than $\sim 50\%$ for ρ and η (with the exception of the W^+W^- channel, where the effect of the exchange of η is larger), while for Δ one can have an enhancement of order 2 – 3. For $m_\Phi \sim 1.5$ TeV and $a_\Phi \sim 1$, the exchange of ρ can enhance WZ by a factor of 3, while the exchange of η can enhance W^+W^- and hh by up to a factor of 5 and 3 respectively. For the same mass and coupling, the exchange of Δ enhances W^+W^+ by a factor of 11 and WZ by a factor of 9.

A consequence of these results is that only for scalar resonances below ~ 1.5 TeV and $a_\Phi \gtrsim 1$ can one hope to successfully study hh production before the luminosity upgrade of the LHC. Of course it all crucially depends on the value of the Higgs mass. But that should become known in a very short time.

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Appendix

A Generators of $SO(5)$

We report here the expression of the $SO(5)$ generators adopted in our work. We define as follows the generators of $SO(5) \rightarrow SO(4)'$, where $SO(4)'$ is the gauged subgroup:

$$\begin{aligned} T_{ij}^{\hat{a}}(0) &= \frac{i}{\sqrt{2}} (\delta^{\hat{a}i} \delta^{5j} - \delta^{\hat{a}j} \delta^{5i}) , \\ T_{ij}^{a\,L/R}(0) &= -\frac{i}{2} \left(\frac{1}{2} \epsilon^{abc} (\delta^{bi} \delta^{cj} - \delta^{bj} \delta^{ci}) \pm (\delta^{ai} \delta^{4j} - \delta^{aj} \delta^{4i}) \right) . \end{aligned} \quad (86)$$

Here $\hat{a} = 1, 2, 3, 4$, $a, b, c = 1, 2, 3$ and $i, j = 1, \dots, 5$. The generators of $SO(5) \rightarrow SO(4)$, denoted as $T^\alpha(\theta)$ in the text ($\alpha = a, \hat{a}$), can be obtained by rotating the $T^\alpha(0)$ by an angle θ :

$$T^\alpha(\theta) = r(\theta) T^\alpha(0) r^{-1}(\theta) , \quad r(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \theta & -\sin \theta \\ 0 & 0 & 0 & \sin \theta & \cos \theta \end{pmatrix} . \quad (87)$$

In particular, one has ($i, \hat{i} = 1, 2, 3$):

$$\begin{aligned} T_{L/R}^i(\theta) &= \frac{T_L^i(0) + T_R^i(0)}{2} \pm \cos \theta \frac{T_L^i(0) - T_R^i(0)}{2} \pm \frac{\sin \theta}{\sqrt{2}} T^{\hat{i}}(0) \\ T^{\hat{i}}(\theta) &= -\sin \theta \frac{T_L^i(0) - T_R^i(0)}{\sqrt{2}} + \cos \theta T^{\hat{i}}(0) \\ T^{\hat{4}}(\theta) &= T^{\hat{4}}(0) . \end{aligned} \quad (88)$$

With the above definitions, all the generators are normalized as $\text{Tr}[T^\alpha T^\beta] = \delta^{\alpha\beta}$. They satisfy the following commutation rules:

$$\begin{aligned}
[T^{aL}, T^{bR}] &= 0, & [T^{aL}, T^{bL}] &= i\epsilon^{abc}T^{cL}, & [T^{aR}, T^{bR}] &= i\epsilon^{abc}T^{cR} \\
[T^{\hat{i}}, T^{\hat{4}}] &= \frac{i}{2}\delta^{\hat{i}k}(T^{kL} - T^{kR}), & [T^{\hat{i}}, T^{\hat{j}}] &= \frac{i}{2}\epsilon^{\hat{i}\hat{j}k}(T^{kL} + T^{kR}) \\
[T^{\hat{i}}, T^{aL}] &= -\frac{i}{2}\delta^{\hat{i}a}T^{\hat{4}} + \frac{i}{2}\epsilon^{\hat{i}a\hat{j}}T^{\hat{j}}, & [T^{\hat{i}}, T^{aR}] &= \frac{i}{2}\delta^{\hat{i}a}T^{\hat{4}} + \frac{i}{2}\epsilon^{\hat{i}a\hat{j}}T^{\hat{j}} \\
[T^{\hat{4}}, T^{aL}] &= \frac{i}{2}\delta^{a\hat{i}}T^{\hat{i}}, & [T^{\hat{4}}, T^{aR}] &= -\frac{i}{2}\delta^{a\hat{i}}T^{\hat{i}}.
\end{aligned} \tag{89}$$

From the above relations one can extract the $SO(5)$ structure constants, $C^{\alpha\beta\gamma}$, which are defined by $[T^\alpha, T^\beta] = iC^{\alpha\beta\gamma}T^\gamma$. They satisfy the following identities (an implicit sum over repeated indices is understood):

$$\begin{aligned}
C^{\hat{a}\hat{b}a_L}C^{\hat{c}\hat{d}a_L} &= \frac{1}{2}C^{\hat{a}\hat{b}e}C^{\hat{c}\hat{d}e} - \frac{1}{4}\epsilon^{\hat{a}\hat{b}\hat{c}\hat{d}} \\
C^{\hat{a}\hat{b}a_R}C^{\hat{c}\hat{d}a_R} &= \frac{1}{2}C^{\hat{a}\hat{b}e}C^{\hat{c}\hat{d}e} + \frac{1}{4}\epsilon^{\hat{a}\hat{b}\hat{c}\hat{d}},
\end{aligned} \tag{90}$$

where

$$C^{\hat{a}\hat{b}e}C^{\hat{c}\hat{d}e} = -\frac{1}{2}(\delta^{ad}\delta^{cb} - \delta^{ac}\delta^{bd}). \tag{91}$$

Finally, the following identities and Fierz relations hold for the $SO(5)$ generators ($T^{\hat{a}}(\theta)$, $T^a(\theta)$ respectively denote a broken and an unbroken generator):

$$\begin{aligned}
(T^a(0)T^b(0))_{55} &= 0 \\
(T^a(0)T^{\hat{b}}(0))_{55} &= (T^{\hat{a}}(0)T^b(0))_{55} = 0 \\
(T^{\hat{a}}(0)T^{\hat{b}}(0))_{55} &= \frac{\delta^{\hat{a}\hat{b}}}{2} = \frac{1}{2}\text{Tr}[T^{\hat{a}}(\theta)T^{\hat{b}}(\theta)],
\end{aligned} \tag{92}$$

$$\text{Tr}(T^{\hat{a}}T^{\hat{b}}T^{\hat{c}}T^{\hat{d}}) = \frac{1}{4}\text{Tr}(T^{\hat{a}}T^{\hat{b}})\text{Tr}(T^{\hat{c}}T^{\hat{d}}) + \frac{1}{4}\text{Tr}(T^{\hat{a}}T^{\hat{d}})\text{Tr}(T^{\hat{c}}T^{\hat{b}}) \tag{93}$$

$$\text{Tr}(T^a\{T^{\hat{a}}, T^{\hat{b}}\}) = 0 \tag{94}$$

for $\hat{a}, \hat{b}, \hat{c}, \hat{d} = 1, 2, 3, 4$ and $a = a_L, a_R, b = b_L, b_R$;

$$\begin{aligned}
\sum_{\alpha=\hat{a}, a_L, a_R} (T^\alpha)_{ij} (T^\alpha)_{kl} &= \frac{1}{2}(\delta^{il}\delta^{jk} - \delta^{ik}\delta^{jl}), \\
\sum_{a_L} (T^{a_L})_{ij} (T^{a_L})_{kl} - \sum_{a_R} (T^{a_R})_{ij} (T^{a_R})_{kl} &= -\frac{1}{2}\epsilon^{ijkl5}, \\
\sum_{a_L} (T^{a_L})_{ij} (T^{a_L})_{kl} + \sum_{a_R} (T^{a_R})_{ij} (T^{a_R})_{kl} &= -\frac{1}{4}\epsilon^{ijmn5}\epsilon^{klmn5},
\end{aligned} \tag{95}$$

for $i, j, k, l, m, n = 1, \dots, 5$.

B Chiral lagrangian and identities for \mathcal{G}/\mathcal{H} symmetric

The quotient space \mathcal{G}/\mathcal{H} is said to be symmetric if there exists an automorphism of the algebra (grading), R , under which the broken generators change sign. Notable examples are $SO(5)/SO(4)$, as discussed in section 2.1, as well as $SO(4)/SO(3)$ and $SU(n) \times SU(n)/SU(n)$. As originally noticed in [13], in the case of a symmetric \mathcal{G}/\mathcal{H} the field $\bar{\Sigma} \equiv U^2 = \exp(2i\Pi(x))$ transforms linearly under global transformations $g \in \mathcal{G}$. This can be easily shown by acting on eq.(4) with the grading R and taking the hermitian conjugate; one finds: $U(\Pi) \rightarrow h(g, \Pi)U(\Pi)R(g)^\dagger$, where $R(g)$ is the element of \mathcal{G} obtained by acting on g with R . Combining this rule with that of eq.(4) one finds:

$$\bar{\Sigma} \equiv U(\Pi)^2, \quad \bar{\Sigma} \rightarrow g \bar{\Sigma} R(g)^\dagger. \quad (96)$$

Hence $\bar{\Sigma}$ transforms linearly under \mathcal{G} , $R(g)$ being a *global* element of \mathcal{G} with no dependence upon the NG field $\Pi(x)$.

A derivative of $\bar{\Sigma}$ covariant under local \mathcal{G} transformation is defined as

$$D_\mu \bar{\Sigma} \equiv \partial_\mu \bar{\Sigma} + iA_\mu \bar{\Sigma} - i\bar{\Sigma} A_\mu^{(R)}, \quad (97)$$

where $A_\mu = A_\mu^a T^a + A_\mu^{\hat{a}} T^{\hat{a}}$ is the external gauge field and $A_\mu^{(R)}$ is obtained by acting on A_μ with the grading: $A_\mu^{(R)} \equiv R[A_\mu] = A_\mu^a T^a - A_\mu^{\hat{a}} T^{\hat{a}}$. Under a (local) transformation $g \in \mathcal{G}$, the gauge field transforms as follows:

$$\begin{aligned} A_\mu &\rightarrow g A_\mu g^\dagger + i(\partial_\mu g)g^\dagger \\ A_\mu^{(R)} &\rightarrow R(g)A_\mu^{(R)}R(g)^\dagger + i(\partial_\mu R(g))R(g)^\dagger. \end{aligned} \quad (98)$$

The following identities thus allow one to rewrite the CCWZ covariant variables d_μ , $E_{\mu\nu}$ in terms of $\bar{\Sigma}$:

$$\begin{aligned} d_\mu &= -\frac{i}{2} U^\dagger (D_\mu \bar{\Sigma}) U^\dagger = +\frac{i}{2} U (D_\mu \bar{\Sigma})^\dagger U \\ E_{\mu\nu} &= -\frac{i}{4} U^\dagger (D_\mu \bar{\Sigma} D_\nu \bar{\Sigma}^\dagger - D_\nu \bar{\Sigma} D_\mu \bar{\Sigma}^\dagger) U + \frac{1}{2} (U^\dagger A_{\mu\nu} U + U A_{\mu\nu}^{(R)} U^\dagger) = -i[d_\mu, d_\nu] + f_{\mu\nu}^+. \end{aligned} \quad (99)$$

$$(100)$$

In the last equality we used the fact that, for a symmetric \mathcal{G}/\mathcal{H} , $f_{\mu\nu}^\pm$ coincide respectively with the even and odd components of $f_{\mu\nu}$ under the grading R :

$$f_{\mu\nu}^\pm = \frac{1}{2} (f_{\mu\nu} \pm f_{\mu\nu}^{(R)}), \quad (101)$$

where $f_{\mu\nu}^{(R)} \equiv R[f_{\mu\nu}] = U A_{\mu\nu}^{(R)} U^\dagger$. Finally, one can prove that:

$$f_{\mu\nu}^- = \nabla_{[\mu} d_{\nu]}. \quad (102)$$

Use of the linear field $\bar{\Sigma}$ suggests an alternative approach to the formulation of the chiral lagrangian, in which this latter is constructed in terms of operators manifestly invariant under global \mathcal{G} transformations. Although this approach, historically adopted for the QCD pion lagrangian, allows one to easily classify and construct all P_{LR} -even operators purely in terms of the field $\bar{\Sigma}$, we find that this is not true in the case of the operators odd under P_{LR} , which are most efficiently and systematically constructed by means of the CCWZ technique.

C Proof of two identities used in the text

We present here the proof of two identities used in the text.

First, we prove eq.(14) and its equivalence with eq.(15). From the definition (11), eq.(9) and the identities (92), it follows:

$$\begin{aligned}
(D_\mu \Phi)^T (D^\mu \Phi) &= \Phi_0^T (D_\mu U)^\dagger (D^\mu U) \Phi_0 \\
&= (r(\theta)^{-1} (D_\mu U)^\dagger (D^\mu U) r(\theta))_{55} \\
&= (r(\theta)^{-1} (d_\mu + E_\mu)^2 r(\theta))_{55} \\
&= \frac{1}{2} \text{Tr}[d_\mu d^\mu] ,
\end{aligned} \tag{103}$$

where $r(\theta)$ is defined in eq.(87). To prove eq.(14) it is convenient to first evaluate the kinetic term of Φ on the $SO(5)/SO(4)$ vacuum, and use as before eqs.(11), (9) and (92):

$$\begin{aligned}
\langle (D_\mu \Phi)^T (D^\mu \Phi) \rangle &= \Phi_0^T (A_\mu^{\hat{a}} T^{\hat{a}}(\theta) + A_\mu^a T^a(\theta))^2 \Phi_0 \\
&= \left(r(\theta)^{-1} (A_\mu^{\hat{a}} T^{\hat{a}}(\theta) + A_\mu^a T^a(\theta))^2 r(\theta) \right)_{55} \\
&= \frac{1}{2} \text{Tr} \left[(A_\mu^{\hat{a}} T^{\hat{a}}(\theta) + A_\mu^a T^a(\theta))^2 \right] \\
&= \frac{1}{4} \sin^2 \theta (W_\mu^+ W^{\mu -} + (W_\mu^3 - B_\mu)^2) ,
\end{aligned} \tag{104}$$

where the last identify follows from eqs.(18) and (19). Then, eq.(14) is obtained by performing a local $SU(2)_L \times SU(2)_R$ rotation which reintroduces the eaten NG bosons χ , and by allowing for $(SU(2)_L \times SU(2)_R)$ -invariant fluctuations $h(x)$ around the vacuum.

A similar strategy can be adopted to prove eq.(27). From eq.(101), in terms of the linear field $\bar{\Sigma}$, the operator O_5^+ can be rewritten as

$$O_5^+ = \frac{1}{2} (\text{Tr}[F_{\mu\nu}^2] - \text{Tr}[\bar{\Sigma}^\dagger F_{\mu\nu} \bar{\Sigma} F_{\mu\nu}^{(R)}]) . \tag{105}$$

On the $SO(5)/SO(4)$ vacuum, for $F_{\mu\nu}^{(R)} = R^\dagger F_{\mu\nu} R$ with R defined in eq.(21), one has

$$\begin{aligned}
\langle O_5^+ \rangle &= \frac{1}{2} (\text{Tr}[F_{\mu\nu}^2] - \text{Tr}[F_{\mu\nu} R^\dagger F_{\mu\nu} R]) \\
&= \frac{1}{2} \sin^2 \theta ((W_\mu^a)^2 + (B_{\mu\nu})^2 - W_{\mu\nu}^3 B^{\mu\nu}) .
\end{aligned} \tag{106}$$

Then, eq.(27) follows by performing a local $SU(2)_L \times SU(2)_R$ rotation to reintroduce the NG bosons χ , and by allowing for $(SU(2)_L \times SU(2)_R)$ -invariant fluctuations $h(x)$ around the vacuum.

D σ -model for the ρ_L vector

As explained in the text, the interactions of the ρ vector at energies above its mass are more easily characterized in terms of the eaten NG bosons. The sigma-model governing their

dynamics is studied by taking a limit in which the transverse polarizations of the ρ decouple ($g_\rho \rightarrow 0$) while the strength of the interactions of its longitudinal polarizations is kept fixed (m_ρ/g_ρ constant). The resulting coset is $SO(5) \times SU(2)_H/SU(2)'_L \times SU(2)_R$ where

$$SU(2)_L \times SU(2)_R \sim SO(4) \subset SO(5), \quad SU(2)'_L = [SU(2)_L \times SU(2)_H]_{\text{diag}}.$$

The coset is parametrized by 7 Goldstone bosons $\pi^{\hat{i}}$ ($\hat{i} = 1, \dots, 4$) and η^a ($a = 1, \dots, 3$) transforming under the unbroken $SU(2)_R \times SU(2)'_L$ respectively as a (2,2) and a (3,1). The ρ vector gauges the extra $SU(2)_H$ factor and eats the three η s.

A convenient parametrization for the NG fields is ²²

$$U = e^{i\pi} e^{i\eta}, \quad \pi = \pi^{\hat{i}} T^{\hat{i}}, \quad \eta = \eta^a X^a, \quad (107)$$

where $T^{\hat{i}}$ are the four $SO(5)/SO(4)$ generators, and $X^a = T_L^a - T'^a$ are the three generators of $SU(2)_L \times SU(2)_H/SU(2)'_L$. Under a transformation $g_5 \in SO(5)$, $g_H \in SU(2)_H$, the NG fields transform as

$$U(\pi, \eta) \rightarrow U(\pi', \eta') = g_5 g_H U h_L'^{\dagger}(\pi, \eta, g_5, g_H) h_R^{\dagger}(\pi, g_5). \quad (108)$$

We thus consider the combination

$$U^{\dagger} \partial_{\mu} U = e^{-i\eta} e^{-i\pi} \partial_{\mu} e^{i\pi} e^{i\eta} + e^{-i\eta} \partial_{\mu} e^{i\eta} = e^{-i\eta} (i d_{\mu} + i E_{\mu}) e^{i\eta} + e^{-i\eta} \partial_{\mu} e^{i\eta}. \quad (109)$$

which transforms as

$$U^{\dagger} \partial_{\mu} U \rightarrow h_R h_L' U^{\dagger} \partial_{\mu} U h_L'^{\dagger} h_R^{\dagger} - i h_R h_L' \partial_{\mu} (h_R h_L')^{\dagger}. \quad (110)$$

Notice that d_{μ} and E_{μ} are the usual covariant variables for $SO(5)/SO(4)$. Since the broken generators $T^{\hat{i}}$ and X^a transform under two different irreducible representations of the unbroken group, it follows that the coset will contain two independent d symbols, one in the algebra spanned by the generators $T^{\hat{i}}$ and another in the algebra of the X^a . It is clear from the parametrization we are using that the only term in eq.(109) that is parallel to the $T^{\hat{i}}$ is the first one in parenthesis. In particular

$$e^{-i\eta} i d_{\mu} e^{i\eta} = i d_{\mu}^{\hat{i}} R(\eta)^{\hat{i}\hat{j}} T^{\hat{j}}, \quad (111)$$

where $R(\eta)$ is just an η -dependent $SO(4)$ rotation. We can thus add to the lagrangian the term

$$(d_{\mu}^{\hat{i}} R(\eta)^{\hat{i}\hat{j}})^2 = d_{\mu}^{\hat{j}} d^{\hat{\mu}}_{\hat{j}} \quad (112)$$

which contains just the π fields and is the analogous of eq.(15). It is also clear that in (109) only the term

$$e^{-i\eta} (\partial_{\mu} + i E_{\mu}) e^{i\eta} \quad (113)$$

²²Here and in the following equations of this section we absorb the decay constants into the NG fields for simplicity.

has a component in the X^a algebra. We denote this with $E_\mu^{(L)a} T_L^a$. We can systematically expand (113) in terms of the π and η fields and project it on the X^a algebra. We show the first few terms

$$\begin{aligned} \tilde{d}_\mu^a &= \partial_\mu \eta^a - \frac{1}{6} (\delta^{bd} \delta^{ca} - \delta^{ba} \delta^{cd}) \partial_\mu \eta^b \eta^c \eta^d + \mathcal{O}(\eta^5) \\ &\quad + \frac{1}{2} E_\mu^{(L)a} - \frac{1}{2} \epsilon^{abc} E_\mu^{(L)b} \eta^c + \mathcal{O}(\pi^2 \eta^2), \end{aligned} \quad (114)$$

where

$$E_\mu^{(L)a} = \frac{1}{4} \epsilon^{abc} \pi^b \partial_\mu \pi^c + \frac{1}{4} (\pi^a \partial_\mu \pi^4 - \pi^4 \partial_\mu \pi^a) + \mathcal{O}(\pi^4). \quad (115)$$

E Scattering amplitudes

In this Appendix we collect the expressions of the scattering amplitudes among longitudinally polarized vector bosons used in our analysis. By means of the Equivalence Theorem these can be derived, at leading order in (m_W/E) , from the scattering amplitudes among $SO(5)/SO(4)$ NG bosons defined in eq.(29):

$$\begin{aligned} \mathcal{A}(W_L^+ W_L^- \rightarrow Z_L Z_L) &\simeq A(s, t, u) \\ \mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &\simeq A(s, t, u) + A(t, s, u) \\ \mathcal{A}(Z_L Z_L \rightarrow Z_L Z_L) &\simeq A(s, t, u) + A(t, s, u) + A(u, t, s) \\ \mathcal{A}(W_L^\pm Z_L \rightarrow W_L^\pm Z_L) &\simeq A(t, s, u) \\ \mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) &\simeq A(t, s, u) + A(u, t, s) \\ \mathcal{A}(W_L^+ W_L^- \rightarrow hh) &\simeq A(s, t, u) \\ \mathcal{A}(Z_L Z_L \rightarrow hh) &\simeq A(s, t, u) \\ \mathcal{A}(W_L^+ W_L^- \rightarrow Zh) &\simeq B(s, t, u). \end{aligned} \quad (116)$$

In terms of the $SO(3)$ isospin amplitudes

$$\begin{aligned} T(0) &= 3A(s, t, u) + A(t, s, u) + A(u, t, s) \\ T(1) &= A(t, s, u) - A(u, t, s) \\ T(2) &= A(t, s, u) + A(u, t, s) \\ T'(0) &= \sqrt{3}A(s, t, u), \end{aligned} \quad (117)$$

one has:

$$\begin{aligned}
\mathcal{A}(W_L^+ W_L^- \rightarrow Z_L Z_L) &= \frac{1}{3}[T(0) - T(2)] \\
\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= \frac{1}{6}[2T(0) + 3T(1) + T(2)] \\
\mathcal{A}(Z_L Z_L \rightarrow Z_L Z_L) &= \frac{1}{3}[T(0) + 2T(2)] \\
\mathcal{A}(W_L^\pm Z_L \rightarrow W_L^\pm Z_L) &= \frac{1}{2}[T(1) + T(2)] \\
\mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) &= T(2) \\
\mathcal{A}(W_L^+ W_L^- \rightarrow hh) &= \frac{1}{\sqrt{3}}T'(0).
\end{aligned} \tag{118}$$

F Operators of the ρ effective lagrangian

We report here the full list of independent operators that must be included in the effective lagrangian of the ρ at leading order in ∂/Λ , that contribute to scattering processes involving up to 2 ρ vectors. According to the criterion of PUVc, they give a subleading contribution to $\pi\pi \rightarrow \pi\pi$ scattering at high energy. In addition to those already defined in eq.(33) and to the mass and kinetic terms of eq.(32) we find:

OPERATORS WITH 1 ρ

$$Q_5 = \text{Tr} [\bar{\rho}^\mu [\nabla_\mu d_\nu, d^\nu]] \quad Q_6 = \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\bar{\rho}_\mu \nabla_\nu d_\rho d_\sigma] \tag{119a}$$

OPERATORS WITH 2 ρ

$$Q_{1(2)} = \text{Tr} [(\nabla_\mu \bar{\rho}^\mu)^2] \quad Q_{7(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}^\mu d_\nu d^\nu] \tag{120a}$$

$$Q_{2(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu f_{\mu\nu}^+] \quad Q_{8(2)} = \text{Tr} [\bar{\rho}_\mu d_\nu \bar{\rho}^\mu d^\nu] \tag{120b}$$

$$Q_{3(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}^\mu] \text{Tr} [d_\mu d^\mu] \quad Q_{9(2)} = \text{Tr} [\bar{\rho}_\mu d^\mu \bar{\rho}_\nu d^\nu] \tag{120c}$$

$$Q_{4(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu] \text{Tr} [d^\mu d^\nu] \quad Q_{10(2)} = \text{Tr} [\bar{\rho}_\mu d_\nu \bar{\rho}^\nu d^\mu] \tag{120d}$$

$$Q_{5(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu \{d^\mu, d^\nu\}] \quad Q_{11(2)} = \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu d_\rho d_\sigma] \tag{120e}$$

$$Q_{6(2)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu \{d^\mu, d^\nu\}] \quad Q_{12(2)} = \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\bar{\rho}_\mu d_\nu \bar{\rho}_\rho d_\sigma] \tag{120f}$$

OPERATORS WITH 3 ρ

$$Q_{1(3)} = \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu \rho^{\mu\nu}] \quad Q_{3(3)} = \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\bar{\rho}_\mu \bar{\rho}_\nu \rho_{\rho\sigma}] \tag{121a}$$

$$Q_{2(3)} = \text{Tr} [\nabla_\mu \rho_\nu \{\bar{\rho}^\mu, \bar{\rho}^\nu\}] \tag{121b}$$

In order to eliminate other linearly dependent operators we made use of the relations reported in Appendix A and B as well as of the following identity:

$$\rho_{\mu\nu} = \nabla_{[\mu} \bar{\rho}_{\nu]} + E_{\mu\nu} + i[\bar{\rho}_\mu, \bar{\rho}_\nu]. \tag{122}$$

Notice that since the grading of $SO(5)/SO(4)$ is an internal automorphism, $R \subset SO(4)$, all operators involving an odd number of d symbols vanish.

Assuming the validity of the derivative expansion up to a scale $\Lambda = g_* f$ and requiring all interactions to remain weaker than g_* at the scale Λ (see section 3.1), the coefficient of each of the above operators is bounded to be $\ll 1/g_*^2$. This implies in particular that the contribution of $Q_{1(2)}$, Q_5 , Q_6 , the only operators that modify $\pi\pi$ scattering, is subdominant. Finally, none of the above operators contribute to any low-energy observable at $\mathcal{O}(p^4)$. This follows from the equation of motions of the vector resonance: $\bar{\rho}_\mu = \rho_\mu - E_\mu \propto \nabla_\nu E_{\nu\mu}$.

G $SO(5)/SO(4)$ linearized models

In this Appendix we define the models LMCHM4 and LMCHM5, which were considered in section 4.1 to check the validity of our analytic approximation. They respectively correspond to the linearized versions of the Minimal Composite Higgs Models MCHM4 and MCHM5 of Refs.[2], [16]. The LMCHM4, in particular, has been previously introduced in Ref.[19]. The two models are defined by the lagrangians

$$\mathcal{L}_{4(5)} = \frac{1}{2}(D_\mu \varphi)^T D^\mu \varphi - \lambda(\varphi^T \varphi - f_0^2)^2 - V_{4(5)}(\vec{\varphi}, \varphi_5) \quad (123)$$

in terms of a scalar multiplet φ transforming in the fundamental representation of $SO(5)$. The potential term $V_{4(5)}$, the only one which differentiates the LMCHM4 from the LMCHM5, breaks explicitly the $SO(5)$ symmetry and triggers the EWSB. In the original models it arises from the one-loop contribution of the SM fermion and gauge fields. Here, for simplicity, we take $V_{4(5)}$ to match the leading term in the expansion of the logarithm that appears in the Coleman-Weinberg potential of Refs. [2, 16].²³

The field φ can be redefined as

$$\varphi(x) = \bar{\eta}(x)\Phi(x) = \bar{\eta}(x)U(x)\Phi_0, \quad (124)$$

where $U(x)$ is defined as in eq.(11) upon identifying $f \equiv \langle \bar{\eta} \rangle$. From eq.(103) it thus follows

$$\mathcal{L}_{4(5)} = \frac{1}{2}\partial_\mu \bar{\eta} \partial^\mu \bar{\eta} + \frac{\bar{\eta}^2}{4} \text{Tr}[d_\mu d^\mu] - \lambda(\bar{\eta}^2 - f_0^2)^2 - V_{4(5)}(\bar{\eta}, h). \quad (125)$$

The derivative couplings that follow from the above lagrangian, after redefining $\bar{\eta} = \eta + \langle \bar{\eta} \rangle$, are of the form of those in eq.(55) with $a_\eta = 1 = b_\eta$. The expression of the potential term $V_{4(5)}$ is explicitly reported in the following for each of the two models.

LMCHM4

In the LMCHM4 the symmetry breaking potential is given by

$$V_4 = \alpha f_0^3 \varphi_5 - \beta f_0^2 \vec{\varphi}^2, \quad (126)$$

²³As noticed by Refs. [2, 16], this approximation is quite good thanks to the fast convergence of the one-loop integral.

where α and β are two dimensionless free parameters. Using eq.(12), one can rewrite V_4 as:

$$V_4 = \alpha f_0^3 \bar{\eta}(x) \cos\left(\theta + \frac{h(x)}{f}\right) - \beta f_0^2 \bar{\eta}^2 \sin^2\left(\theta + \frac{h(x)}{f}\right). \quad (127)$$

The expectation value $\langle \bar{\eta} \rangle = f$ and θ are thus given by

$$\langle \eta \rangle^2 \equiv f^2 = f_0^2 \left(1 + \frac{\beta}{2\lambda}\right) \equiv f_0^2(1+z), \quad \cos \theta = -\frac{\alpha}{2\beta} \frac{1}{\sqrt{1+z}} \equiv \sqrt{1-\xi}, \quad (128)$$

with $z \equiv \beta/2\lambda$. By expanding the potential around the vacuum one gets the scalar mass matrix

$$\mathcal{M} = \begin{pmatrix} m_{\eta\eta}^2 & m_{h\eta}^2 \\ m_{h\eta}^2 & m_{hh}^2 \end{pmatrix}, \quad (129)$$

with

$$\begin{aligned} m_{\eta\eta}^2 &= 8\lambda f^2 \frac{1+3/2z}{1+z} - m_{hh}^2, \\ m_{\eta h}^2 &= -m_{hh}^2 \frac{\sqrt{1-\xi}}{\sqrt{\xi}}, \\ m_{hh}^2 &= \frac{2\beta v^2}{1+z}. \end{aligned} \quad (130)$$

Working in the small mixing limit, $m_h \ll m_\eta$, one can easily derive from the potential the non-derivative hhh and $hh\eta$ couplings, the only ones relevant to our analysis:

$$\begin{aligned} V_4(h, \eta) &\supset \frac{g_{hhh}}{6} h^3 + \frac{g_{hh\eta}}{2} h^2 \eta, \\ g_{hhh} &\approx 3 \frac{m_h^2}{v} \sqrt{1-\xi}, \\ g_{hh\eta} &\approx -\frac{m_h^2}{v} \frac{1-3\xi}{\sqrt{\xi}}. \end{aligned} \quad (131)$$

LMCHM5

In the case of the LMCHM5 the potential has the form

$$V_5 = \alpha f_0^2 \bar{\varphi}^2 - \beta \varphi_5^2 \bar{\varphi}^2, \quad (132)$$

which can be rewritten as:

$$V_5 = \alpha f_0^2 \bar{\eta}^2 \sin^2\left(\theta + \frac{h(x)}{f}\right) - \beta \bar{\eta}^4 \sin^2\left(\theta + \frac{h(x)}{f}\right) \cos^2\left(\theta + \frac{h(x)}{f}\right). \quad (133)$$

By minimizing the potential, one obtains:

$$\langle \eta \rangle^2 \equiv f^2 = f_0^2 \left(\frac{\alpha - 4\lambda}{\beta - 4\lambda}\right) \equiv f_0^2(1+z), \quad \sin^2 \theta = \frac{2\lambda}{\beta} \frac{\alpha - \beta}{\alpha - 4\lambda} \equiv \xi, \quad (134)$$

with $z \equiv (\alpha - \beta)/(\beta - 4\lambda)$. The scalar masses are given by

$$\begin{aligned}
m_{\eta\eta}^2 &= 8\lambda f^2 \frac{1-z}{1+z} - m_{hh}^2 \frac{\xi}{1-\xi}, \\
m_{\eta h}^2 &= -m_{hh}^2 \frac{1-2\xi}{\sqrt{\xi(1-\xi)}}, \\
m_{hh}^2 &= 8\beta v^2(1-\xi).
\end{aligned} \tag{135}$$

In the small mixing limit the relevant scalar couplings are

$$\begin{aligned}
g_{hhh} &\simeq 3 \frac{m_h^2}{v} \frac{\sqrt{1-2\xi}}{\sqrt{1-\xi}}, \\
g_{hh\eta} &\simeq -\frac{m_h^2}{v} \frac{1-12(1-\xi)\xi}{2(1-\xi)\sqrt{\xi}}.
\end{aligned} \tag{136}$$

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